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Fault Detection for a One-Dimensional Wave PDE System

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Abstract

In this paper, we design a fault diagnosis scheme for a class of variable coefficient wave equation (an overhead crane system), which is composed of an observer and its output error is treated as residual signal. When the system is in a healthy state, the output residual signal decays exponentially. Due to the existence of external disturbance, the residual is not zero in the state without fault. Therefore, we further design a reasonable threshold which is based on the upper bound of the residual dynamics and external disturbance to reduce the influence of external disturbance and determine whether the system fault occurs. The convergence properties of partial differential equation (PDE) observer and residual signal are analyzed by Lyapunov stability theory. Finally, the effectiveness of this fault diagnosis is illustrated by simulation and we can judge whether this overhead crane system fails by this fault diagnosis scheme.

Keywords

Wave Equation, Fault Detection, Observer, Lyapounov Function, Disturbance

1. Introduction

System failure refers to the situation where the system has made mistakes and thus the system cannot work properly. In recent years, with the further development of partial differential equations (PDEs), the distribution parameter systems have been widely used in various industries, such as signal processing [1], manufacturing and chemical processes [2], aerospace [3], and complex biological systems [4], to name just a few. When the system fails to achieve the expected results due to the failure, it will cause huge economic and property losses and even human casualties. Therefore, the detection of system faults is crucial.

According to the location of fault, fault detection can be divided into actuator fault detection [5] [6], sensor fault detection [7] [8] and plant fault detection.

Fault detection methods can be mainly divided into two categories [9]: model-free fault detection method [10] [11] [12] and model-based fault detection method. In general, the model-based fault detection method judges the system state by the residual signal generated by the system. Firstly, a non-zero and constant threshold is designed according to the external disturbance and the state of the system itself, then the residual signal of the system output is obtained. When the residual error of the system output exceeds the designed threshold, the fault of the system detected, otherwise, the system is in a healthy state. The design of threshold can reduce the false alarm rate and make the system robust. This paper mainly studies the model-based plant fault detection problem.

The control and estimation of PDEs have attracted much attention in recent years. There are two main control and estimation methods at present: 1) early lumping: ordinary differential equations (ODEs) are used to approximate PDEs in finite dimensional space [13]. 2) Late lumping: the design is carried out in infinite dimensional space but is implemented in the form of approximate ODEs [14]. Compared with control and estimation, fault detection research is relatively less, but various fields including cloud computing [15] have begun to gradually design and use fault diagnosis schemes. The early lumping method greatly expands the research method of fault detection [16] [17], but finite dimensional approximation often reduces the accuracy of the original PDEs model, and sometimes leads to the neglect of some high-order but important modes in the system. As for research on late lumping, the operator theory [18] [19] and Lyapunov method [20] [21] [22] [23] are wildly applied in fault detection. At the meantime, most of the researches on fault diagnosis based on Lyapunov method focus on parabolic equation, while the researches on fault diagnosis of hyperbolic equation are relatively few. On this basis, this paper will further study the fault detection of a class of one-dimensional variable coefficient wave equation.

In summary, this paper provides a fault detection scheme of variable coefficient wave equation and accomplishes the following objectives 1) the scheme uses the Lyapunov method, relative to the operator theory is more concise, more convenient for engineers to understand and apply. 2) Avoid the use of early block method, reduce the system error. 3) This paper considers the impact of external interference on the system scheme and has practical feasibility.

In this paper, we use the following notation: $\|f\| = \left(\int_0^1 f^2(x)\,\mathrm{d}x\right)^{\frac{1}{2}}$, $e_t = \partial e/\partial t$, $e_s = \partial e/\partial s$, $e_{tt} = \partial^2 e/\partial t^2$, $e_{ss} = \partial^2 e/\partial s^2$. For $a, b, \lambda \in R$, with $\lambda > 0$, the following Young inequality

$$ab \le \frac{\lambda}{2}a^2 + \frac{1}{2\lambda}b^2, ab \ge -\frac{\lambda}{2}a^2 - \frac{1}{2\lambda}b^2,$$

Cauchy-Schwarz inequality

$$\int_0^1 f_1(x) f_2(x)\,\mathrm{d}x \le \|f_1(x)\|\|f_2(x)\|$$

hold.

The paper is organized as follows. In section 2, we give the assumptions for the considered ordinary differential equation—partial differential equation (ODE-PDE) model. A fault detection scheme is proposed in section 3. In section 4, we use numerical simulation to illustrate the effectiveness of fault diagnosis methods. Section 5 concludes the work.

2. Problem Statement

In this paper, we consider the cascade ODE-PDE system as follows.

$$\begin{cases} y_{tt}(s,t) = \left(a(s)y_s(s,t)\right)_s + f(s,t) + d(s,t), \\ y_s(0,t) = 0, \\ y(1,t) = X_p(t), \\ \dot{X}_p = v = U(t), \end{cases} \tag{2.1}$$

with

$$a(s) = gs + \frac{gm}{\rho} \tag{2.2}$$

this system called "overhead crane" in [24] [25]. It is composed of a platform moving along a horizontal step and a flexible cable with a length of 1 attached to a mobile platform. The flexible cable is connected to carry goods of mass m. In this system $t \in [0,\infty]$ is time and $s \in [0,1]$ denotes the arc length along the cable, $y(s,t)$ denotes the horizontal displacement of the point with s in the transverse coordinates of the curve at t time, X_p is the horizontal coordinate of the platform, $y_s(s,t)$ denotes the vertical angle of the flexible cable at s time, ρ denotes the mass of the unit length of the cable, v represents the speed of the platform. g is the acceleration of gravity. $U(t)$ is a control input. The terms $d(x,t)$ and $f(s,t)$ represent an unknown disturbance and an unknown fault, respectively.

Assumption 2.1. The establishment of the system requires the following assumptions

1) The cable is completely flexible and not open.

2) The lateral and angular displacements are small.

3) The acceleration of load mass is independent of gravity acceleration g.

Assumption 2.2. The fault $f(s,t)$ and the disturbance $d(s,t)$ are bounded: $|f| \le \overline{f}$, $|g| \le \overline{g}$. Furthermore, the bound $\overline{f}$, $\overline{g}$ is known.

3. Fault Detection Scheme

In this section, we derive the fault detection scheme. Assume that the velocity of 0 end and the displacement of 1 end are measurable. We design the following observer

$$\begin{cases} \hat{y}_{tt}(s,t) = \left(a\hat{y}_s(s,t)\right)_s, \\ \hat{y}_s(0,t) = -c_1\left(y_t(0,t) - \hat{y}_t(0,t)\right), \\ \hat{y}(1,t) = y(1,t), \end{cases} \tag{3.1}$$

where $c_1 > 0$ is a tuned constant parameter.

Then we define the residual

$$r(t) = e(0,t) \tag{3.2}$$

Subtracting (3.1) from (2.1), we obtain the observer error dynamics

$$\begin{cases} e_{tt}(s,t) = \left(a(s)e_s(s,t)\right)_s + f(s,t) + d(s,t), \\ e_s(0,t) = c_1 e_t(0,t), \\ e(1,t) = 0, \end{cases} \tag{3.3}$$

where $e(s,t) = y(s,t) - \hat{y}(s,t)$.

Theorem 3.1. Consider the error dynamics (3.3) and the residual $r(t)$, then

1) When there is no fault and disturbance, *i.e.* $f(s,t) = 0$, $d(s,t) = 0$, the residual $r(t) \to 0$ exponentially as $t \to \infty$.

2) When there is fault or/and disturbance, *i.e.* $f(s,t) \neq 0$ or/and $d(s,t) \neq 0$, the residual signal $r(t)$ will be upper bounded as $t \to \infty$ by $r(t) \leq \sqrt{H/a(0)}$, where $H = \left(1/(\alpha - \gamma D/2)\right)\left(1/\gamma \left\|\bar{f}(s,t) + \bar{d}(s,t)\right\|^2\right)$.

Proof. Consider the following Lyapunov functional candidate for the error dynamics (3.3)

$$V(t) = \frac{1}{2}\int_0^1 \left(e_t^2(s,t) + a(s)e_s^2(s,t)\right)\mathrm{d}s + \varepsilon\int_0^1 (s-2)e_s(s,t)e_t(s,t)\mathrm{d}s, \tag{3.4}$$

where $\varepsilon > 0$ is a small enough positive parameter.

Considering the term inside the integral of the second term on the right hand side of (3.4) and applying Young inequality, we have

$$\begin{aligned} (s-2)e_s(s,t)e_t(s,t) &\geq -\frac{\lambda}{2}(s-2)^2 e_s^2(s,t) - \frac{1}{2\lambda}e_t^2(s,t) \\ &\geq -2\lambda e_s^2(s,t) - \frac{1}{2\lambda}e_t^2(s,t). \end{aligned} \tag{3.5}$$

Let $\lambda = 1/2$, we have

$$(s-2)e_s(s,t)e_t(s,t) \geq -e_s^2(s,t) - e_t^2(s,t), \tag{3.6}$$

which is equivalent to

$$V(t) \geq \left(\frac{1}{2} - \varepsilon\right)\int_0^1 e_t^2(s,t)\mathrm{d}s + \int_0^1 \left(\frac{1}{2}a(s) - \varepsilon\right)e_s^2(s,t)\mathrm{d}s. \tag{3.7}$$

Hence $V(t)$ is a positive Lyapunov functional for $\varepsilon < \min\{1/2, a(0)/2\}$.

Next, the derivative of $V(t)$ can be obtained from (3.4)

$$\begin{aligned} \dot{V}(t) &= \int_0^1 e_t(s,t)e_{tt}(s,t)\mathrm{d}s + \int_0^1 a(s)e_s(s,t)e_{st}(s,t)\mathrm{d}s \\ &+ \varepsilon\int_0^1 (s-2)e_{st}(s,t)e_t(s,t)\mathrm{d}s \\ &+ \varepsilon\int_0^1 (s-2)e_s(s,t)e_{tt}(s,t)\mathrm{d}s. \end{aligned} \tag{3.8}$$

Consider the first term on the right hand side of (3.8). Applying integration by parts and utilizing the Cauchy-Schwarz inequality, one has

$$
\begin{aligned}
&\int_0^1 e_t(s,t)e_{tt}(s,t)\mathrm{d}s \\
&=\int_0^1 e_t(s,t)\big(a(s)e_s(s,t)\big)_s\mathrm{d}s+\int_0^1 e_t(s,t)\big(f(s,t)+d(s,t)\big)\mathrm{d}s \\
&=-a(0)e_s(0,t)e_t(0,t)-\int_0^1 a(s)e_s(s,t)e_{st}(s,t)\mathrm{d}s \\
&\quad+\int_0^1 e_t(s,t)\big(f(s,t)+d(s,t)\big)\mathrm{d}s \\
&\le -a(0)e_s(0,t)e_t(0,t)-\int_0^1 a(s)e_s(s,t)e_{st}(s,t)\mathrm{d}s \\
&\quad+\|e_t(s,t)\|\|f(s,t)+d(s,t)\|.
\end{aligned}
\tag{3.9}
$$

Next, we consider the third term on the right hand side of (3.8). Applying integration by parts, we have

$$
\begin{aligned}
&\varepsilon\int_0^1(s-2)e_{st}(s,t)e_t(s,t)\mathrm{d}s \\
&=2\varepsilon e_t^2(0,t)-\varepsilon\int_0^1(s-2)e_{st}(s,t)e_t(s,t)\mathrm{d}s-\varepsilon\int_0^1 e_t^2(s,t)\mathrm{d}s.
\end{aligned}
\tag{3.10}
$$

Furthermore, we can obtain

$$
\varepsilon\int_0^1(s-2)e_{st}(s,t)e_t(s,t)\mathrm{d}s=\varepsilon e_t^2(0,t)-\frac{\varepsilon}{2}\int_0^1 e_t^2(s,t)\mathrm{d}s.
\tag{3.11}
$$

Consider the fourth term on the right hand side of (3.8)

$$
\begin{aligned}
&\varepsilon\int_0^1(s-2)e_s(s,t)e_{tt}(s,t)\mathrm{d}s \\
&=\varepsilon\int_0^1(s-2)e_s(s,t)\big(a(s)e_s(s,t)\big)_s\mathrm{d}s \\
&\quad+\varepsilon\int_0^1(s-2)e_s(s,t)\big(f(s,t)+d(s,t)\big)\mathrm{d}s \\
&\le\varepsilon\int_0^1(s-2)e_s(s,t)\big(a(s)e_s(s,t)\big)_s\mathrm{d}s \\
&\quad+2\varepsilon\|e_s(s,t)\|\|f(s,t)+d(s,t)\|.
\end{aligned}
\tag{3.12}
$$

Similar to (3.10), apply integration by parts to the first term on the right hand of (3.12) to give

$$
\begin{aligned}
&\varepsilon\int_0^1(s-2)e_s(s,t)\big(a(s)e_s(s,t)\big)_s\mathrm{d}s \\
&=-\frac{\varepsilon}{2}a(1)e_s^2(1,t)+\varepsilon a(0)e_s^2(0,t)-\frac{\varepsilon}{2}\int_0^1 a(s)e_s^2(s,t)\mathrm{d}s \\
&\quad+\frac{\varepsilon}{2}\int_0^1(s-2)a_s(s)e_s^2(s,t)\mathrm{d}s \\
&\le\varepsilon a(0)e_s^2(0,t)-\frac{\varepsilon}{2}\int_0^1 a(s)e_s^2(s,t)\mathrm{d}s \\
&=\varepsilon a(0)c_1^2e_t^2(0,t)-\frac{\varepsilon}{2}\int_0^1 a(s)e_s^2(s,t)\mathrm{d}s
\end{aligned}
\tag{3.13}
$$

By (3.9), (3.11), (3.12) and (3.13), we obtain

$$
\begin{aligned}
\dot V(t)\le&\left(-a(0)c_1+\varepsilon+\varepsilon a(0)c_1^2\right)e_t^2(0,t)-\frac{\varepsilon}{2}\int_0^1\left(e_t^2(s,t)+a(s)e_s^2(s,t)\right)\mathrm{d}s \\
&+\|e_t(s,t)\|\|f(s,t)+d(s,t)\|+2\varepsilon\|e_s(s,t)\|\|f(s,t)+d(s,t)\|.
\end{aligned}
\tag{3.14}
$$

Since ε is small enough, we have $\left(-a(0)c_1+\varepsilon+\varepsilon a(0)c_1^2\right)<0$.
Hence, (3.14) can be rewritten as

$$\dot{V}(t) \le V_1 + V_2, \tag{3.15}$$

where

$$\begin{aligned} V_1 &= -\frac{\varepsilon}{2}\int_0^1 \left(e_t^2(s,t) + a(s)e_s^2(s,t)\right)\mathrm{d}s, \\ V_2 &= \left\|e_t(s,t)\right\|\left\|f(s,t)+d(s,t)\right\| + 2\varepsilon\left\|e_s(s,t)\right\|\left\|f(s,t)+d(s,t)\right\|. \end{aligned} \tag{3.16}$$

Apply Young inequality in (3.5) with $\lambda = 1/2$, we can write

$$(s-2)e_s(s,t)e_t(s,t) \le e_t^2(s,t) + e_s^2(s,t) \tag{3.17}$$

Hence

$$V(t) \le \left(\frac{1}{2}+\varepsilon\right)\int_0^1 e_t^2(s,t)\mathrm{d}s + \left(\frac{1}{2}+\frac{\varepsilon}{a(0)}\right)\int_0^1 a(s)e_s^2(s,t)\mathrm{d}s, \tag{3.18}$$

$$-V(t) \ge -\left(\frac{1}{2}+\varepsilon\right)\int_0^1 e_t^2(s,t)\mathrm{d}s - \left(\frac{1}{2}+\frac{\varepsilon}{a(0)}\right)\int_0^1 a(s)e_s^2(s,t)\mathrm{d}s, \tag{3.19}$$

consider (3.16) and (3.19)

$$V_1 \le -\alpha V(t), \tag{3.20}$$

where $\alpha = \min\left\{\varepsilon/1+2\varepsilon, \varepsilon a(0)/a(0)+2\varepsilon\right\}$.

Next, we can find the upper bound of V_2

$$\begin{aligned} V_2 &= \left\|e_t(s,t)\right\|\left\|f(s,t)+d(s,t)\right\| + 2\varepsilon\left\|e_s(s,t)\right\|\left\|f(s,t)+d(s,t)\right\| \\ &\le \frac{\gamma}{2}\left(\left\|e_t(s,t)\right\|^2 + \left\|e_s(s,t)\right\|^2\right) + \frac{1}{\gamma}\left\|f(s,t)+d(s,t)\right\|^2 \\ &\le \frac{\gamma}{2}DV + \frac{1}{\gamma}\left\|f(s,t)+d(s,t)\right\|^2, \end{aligned} \tag{3.21}$$

where $D = \max\left\{2/1-2\varepsilon, 2/a(0)-2\varepsilon\right\}$ and $\gamma < 2\alpha/D$.

Finally, using (3.20) and (3.21), we can rewritten (3.15) as

$$\dot{V}(t) \le \left(-\alpha + \frac{\gamma D}{2}\right)V(t) + \frac{1}{\gamma}\left\|f(s,t)+d(s,t)\right\|^2. \tag{3.22}$$

When there is no fault and disturbance, *i.e.* $f(s,t)=0$, $d(s,t)=0$, (3.22) can be rewritten as

$$V(t) \le \mathrm{e}^{-\alpha t}V(0) \tag{3.23}$$

Because $e(1,t) - e(0,t) = \int_0^1 e_s(s,t)\mathrm{d}s$ and $e(1,t)=0$ *i.e.*

$$e^2(0,t) \le \left(\int_0^1 e_s(s,t)\mathrm{d}s\right)^2 \le \int_0^1 e_s^2(s,t)\mathrm{d}s, \tag{3.24}$$

$$a(0)e^2(0,t) \le a(0)\int_0^1 e_s^2(s,t)\mathrm{d}s \le \int_0^1 a(s)e_s^2(s,t)\mathrm{d}s \le V(t). \tag{3.25}$$

Hence, we can know the exponential convergence of $r(t) = e(0,t) \to 0$ as $t \to \infty$ with no fault and disturbance.

When there is fault and/or disturbance, we have

$$\dot{V}(t) \le \left(-\alpha + \frac{\gamma D}{2}\right)V(t) + \frac{1}{\gamma}\left\|f(s,t)+d(s,t)\right\|^2, \tag{3.26}$$

that is

$$V(t) \le \mathrm{e}^{-\alpha+\frac{\gamma D}{2}} V(0) + H, \tag{3.27}$$

where H is defined in Theorem 3.1.

Hence we can conclude that

$$\lim_{t\to\infty} V(t) \le H \tag{3.28}$$

and

$$\begin{aligned} a(0)r^2(t) &= a(0)e^2(0,t) \le a(0)\int_0^1 e_s^2(s,t)\mathrm{d}s \\ &\le \int_0^1 a(s)e_s^2(s,t)\mathrm{d}s \le V(t) \le H. \end{aligned} \tag{3.29}$$

Remark 3.1. The bound H in Theorem is upper bound of the residual signal $r(t)$ under fault and disturbance. In the presence of disturbance but no fault, *i.e.* $f(s,t)=0$ and $d(s,t)\ne 0$, the upper bound of $r(t)$ reduces to

$$|r(t)| \le \sqrt{\frac{\bar{H}}{a(0)}}, \tag{3.30}$$

where $\bar{H} = (1/(\alpha-\gamma D/2))\left((1/\gamma)\|\bar{d}(s,t)\|^2\right)$.

We use this upper bound $\bar{H}$ as a threshold on the residual $r(t)$ with the following fault detection logic: $r(t) > \bar{H} \to$ fault occurred, $r(t) \le \bar{H} \to$ no fault occurred.

4. Simulation Results

In this section, we give a simulation example to illustrate the effectiveness of the above conclusions in this paper. In system (3.3), we choose the following parameters $c_1 = 1$, $g = 9.8$, $m = 1$, $\rho = 1$. The finite element method is adopted to compute the numerical solution of system (3.3) with the time step $\delta = 0.1\,\mathrm{s}$. When there is no fault and disturbance, the solution of error system decays exponentially which is presented in Figure 1. Hence the residual signal $e(0,t)$ tends to 0 exponentially.

Next, we consider the system (3.3) affected by a disturbance $d(s,t) = s\sin t$ and a fault $f(s,t) = 150\sin(10s)\left(1-\mathrm{e}^{-0.3(t-15)}\right)$ (see Figure 2(a)), the fault $f(s,t)$ occurs at time $t = 15\,\mathrm{s}$. The solution of error systems with faults and disturbances is displayed in Figure 3. In this case, the solution of system (3.3) is only bounded but not exponentially stable.

Finally, we select the parameters $\gamma = 1/55$, $\varepsilon = 0.1$ in Theorem 3.1 and (3.4), and calculate the threshold $\bar{H} = 11/\sqrt{3}$ in Remark 3.1. The trajectory of the residual $r(t)$ under the disturbance and the fault is shown in Figure 2(b). As we expected, 1.5 sec after the fault occurs, the residual $r(t)$ crosses the threshold $\bar{H}$.

5. Conclusion

In this paper, we derive and analyze a fault detection scheme for a class of variable coefficients wave equations with external disturbance. The diagnosis scheme detects a fault when the residual signal crosses the given threshold. We also show that the residual signal is bounded in the presence of external disturbance. Furthermore, we illustrate the feasibility of the scheme by a numerical simulation example.

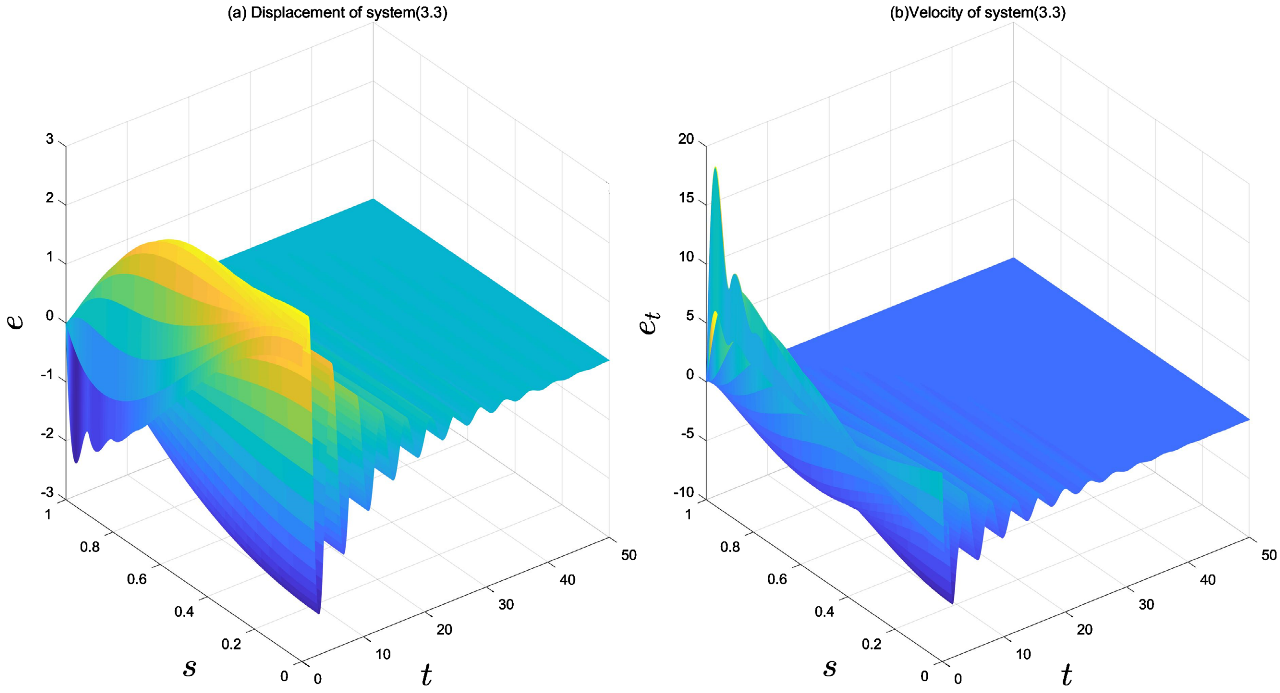

Figure 1. The solution of the system (3.3) without fault and disturbance.

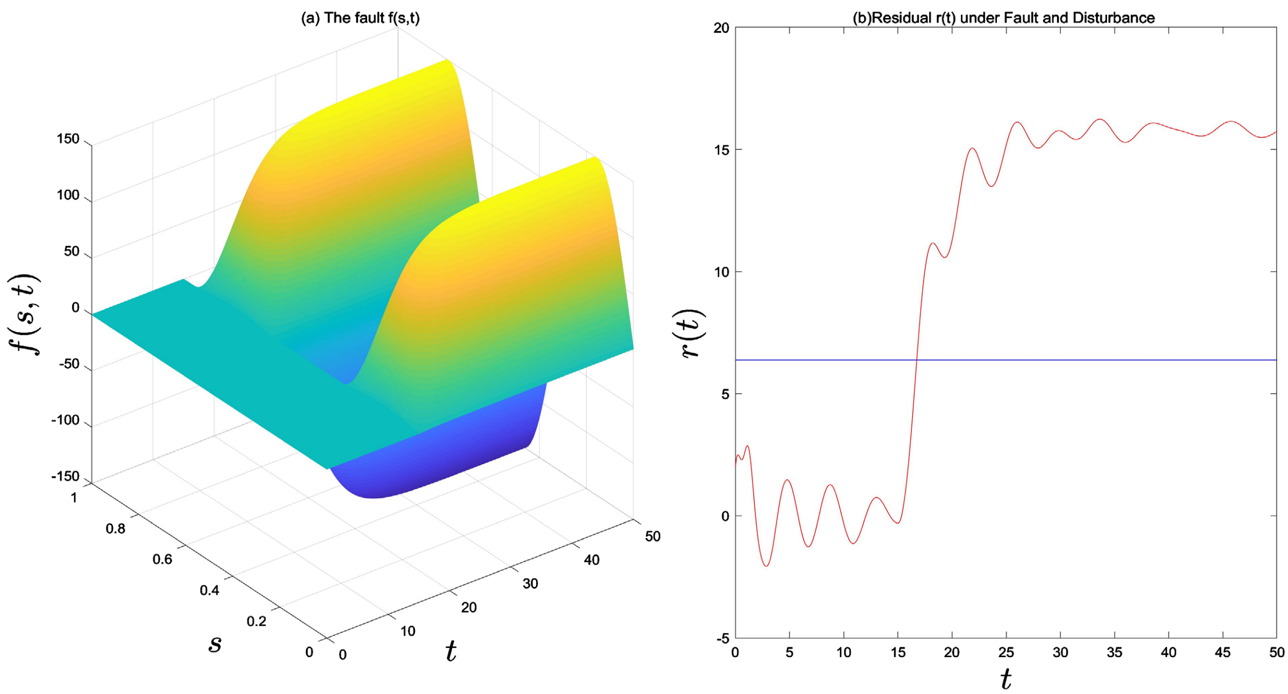

Figure 2. The fault and residual $r(t)$ under fault and disturbance.

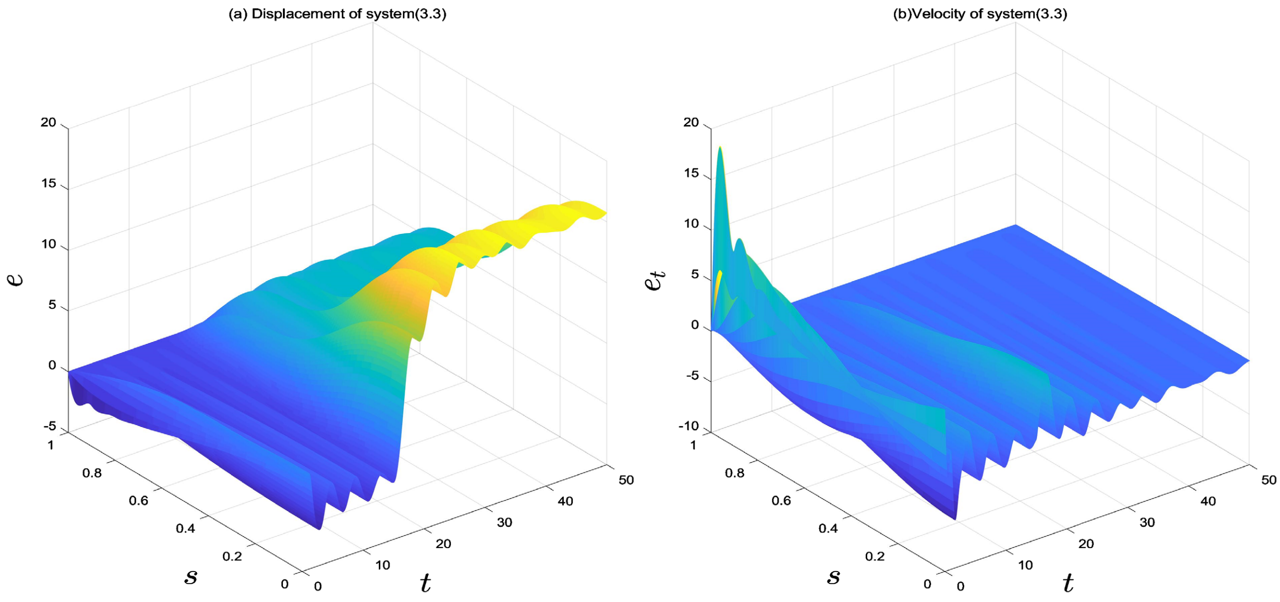

Figure 3. The displacement and velocity of the system (3.3) with fault and disturbance.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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