



Ameliorated Ratio Estimator of Population Mean Using New Linear Combination

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Authors' contributions

This work was carried out in collaboration between both authors. Author TAR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author SM managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AIR/2020/v21i930235

Editor(s):

(1) Dr. Carlos Humberto Martins, State University of Maringá, Brazil.

Reviewers:

(1) Somayeh Ahmadi Goooraji, Tehran University of Medical Sciences (TUMS), Iran.

(2) Ahmed Sami Nori, University of Mosul, Iraq.

Complete Peer review History: <http://www.sdiarticle4.com/review-history/59985>

Original Research Article

Received 05 June 2020
Accepted 10 August 2020
Published 22 August 2020

ABSTRACT

We propose a new modified ratio estimator of population mean of the main variable using the linear combination of known values of Co-efficient of Kurtosis and Tri-Mean of the auxiliary variable. Mean Square Error (MSE) and bias of the proposed estimator is calculated and compared with the existing estimator. The comparison is demonstrated numerically which shows that the proposed estimator performs better than the existing estimators.

Keywords: Estimator; simple random sampling; auxiliary variable; mean square error; bias; co-efficient of Kurtosis; tri-mean.

1. INTRODUCTION

The information on study variable Y is studied with the information on auxiliary variable X . This information on auxiliary variable X , may be

utilized to obtain a more efficient estimator of the population mean. Ratio method of estimation using auxiliary information is an attempt in this direction. The method is utilized under the condition only if the variables are correlated.

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when the population parameters of the auxiliary variable X such as Population Mean, Coefficient of variation, Kurtosis, Skewness, Correlation coefficient, Median etc., are known, a number of estimators such as ratio, product and linear regression estimators and their modifications are available in literature and are performing better than the simple random sample mean under certain conditions. Among these estimators the ratio estimator and its modifications have widely attracted many researchers for the estimation of the mean of the study variable (see for example Kadilar and Cingi [1,2,3], Cingi and Kadilar [4], Subramani [5]). Further improvements are achieved by introducing a large number of modified ratio estimators with the use of known coefficient of variation, kurtosis, skewness, median, coefficient of correlation, Subramani and Kumarpandiyan [6,7,8]. Subzar et al. [9] had taken initiative by proposed modified ratio estimator for estimating the population mean of the study variable by using the population deciles and correlation coefficient of the auxiliary variable.

These points have motivated the authors to propose modified ratio estimators using the above linear combinations. The classical ratio estimator for population mean \bar{Y} is defined as:

$$\hat{Y}_R = \frac{\bar{y}}{\bar{x}} \bar{X} = \hat{R} \bar{X}, \tag{1.1}$$

where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$ and it is assumed that the population mean \bar{X} of the auxiliary variate X is known. Hence \bar{y} is the sample mean of the variate of the interest and \bar{x} is the sample mean of the auxiliary variate. The bias and the mean square error of \hat{Y}_R to the first degree of approximation is given below:

$$Bias(\hat{Y}_R) \cong \frac{1-f}{n} \bar{Y} (C_x^2 - 2C_x C_y \rho) \tag{1.2}$$

$$MSE(\hat{Y}_R) \cong \frac{1-f}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2C_x C_y \rho) \tag{1.3}$$

where, C_x, C_y are the co-efficient of variation and ρ is the co-efficient of correlation.

2. YAN AND TIAN [10] ESTIMATOR

Yan and Tian [10] have proposed two modified ratio estimators using the linear combinations of Co-efficient of skewness and Co-efficient of kurtosis.

$$\hat{Y}_1 = \bar{y} \left[\frac{\bar{X}\beta_2 + \beta_1}{\bar{x}\beta_2 + \beta_1} \right] \tag{2.1}$$

$$\hat{Y}_2 = \bar{y} \left[\frac{\bar{X}\beta_1 + \beta_2}{\bar{x}\beta_1 + \beta_2} \right] \tag{2.2}$$

where, $\beta_1 = \frac{N \sum_{i=1}^N (X_i - \bar{X})^3}{(N-1)(N-2)S^3}$ is Co-efficient of Skewness of auxiliary variable and $\beta_2 = \frac{N(N+1) \sum_{i=1}^N (X_i - \bar{X})^4}{(N-1)(N-2)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$ is Co-efficient of kurtosis. To the first degree of approximation the bias and mean square error are given below:

$$Bias(\hat{Y}_1) = \frac{(1-f)}{n} \bar{Y} (\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho) \tag{2.3}$$

$$MSE(\hat{Y}_1) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho) \tag{2.4}$$

where, $\theta_1 = \frac{\bar{X}\beta_2}{\bar{x}\beta_2 + \beta_1}$

and

$$Bias(\hat{Y}_2) = \frac{(1-f)}{n} \bar{Y} (\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho) \tag{2.5}$$

$$MSE(\hat{Y}_2) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho) \tag{2.6}$$

where, $\theta_2 = \frac{\bar{X}\beta_1}{\bar{x}\beta_1 + \beta_2}$

3. THE SUGGESTED MODIFIED ESTIMATOR

We propose the modified ratio estimator using the linear combination of Co-efficient of Kurtosis and Tri-Mean (TM) of the auxiliary variable. This measure is the weighted average of the population median and two quartiles. For more detailed properties of tri-mean see Wang et al. [11].

$$\hat{Y}_{p1} = \bar{y} \left[\frac{\bar{X}\beta_2 + TM}{\bar{x}\beta_2 + TM} \right] \tag{3.1}$$

where, $\beta_2 = \frac{N(N+1) \sum_{i=1}^N (X_i - \bar{X})^4}{(N-1)(N-2)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Co-efficient of kurtosis and (TM) which is the weighted average of the population median and two quartiles and is defined as $TM = (Q_1 + 2Q_2 + Q_3)/4$,

The bias of \hat{Y}_{p1} upto the first degree of approximation is given by;

$$B(\hat{Y}_{p1}) = \frac{(1-f)}{n} \bar{Y} (\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y \rho), \tag{3.2}$$

Table 1. Parameters and constants of population

<i>N</i>	<i>n</i>	\bar{Y}	\bar{X}	ρ	<i>S_y</i>	<i>C_y</i>	<i>S_x</i>	<i>C_x</i>	β_2	β_1	<i>TM</i>
80	20	51.8264	11.2646	0.9413	18.3569	0.3542	8.4542	0.750	2.866	1.05	9.318

Table 2. Constants, Bias and MSE's of estimators

Estimator	Constants	Bias	MSE
\bar{Y}_1 Yan and Tian [10]	0.968	0.554	881.97
\bar{Y}_2 Yan and Tian [10]	0.804	0.316	539.35
Subramani [5]	0.798	0.352	527.82
Subzar et al. [9]	0.875	0.302	486.32
\hat{Y}_{p1} (Proposed Estimator)	0.776	0.281	396.62

MSE of the above estimator can be found using Taylor series method as :

$$MSE(\hat{Y}_{p1}) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y \rho); \tag{3.3}$$

where, $\theta_{p1} = \frac{\bar{x}\beta_2}{\bar{x}\beta_2 + TM}$

4. COMPARISON OF EFFICIENCY

We compare the efficiencies for which the estimator given in (3.1) \hat{Y}_{p1} is more efficient than the existing estimator;

$$MSE(\hat{Y}_{p1}) < MSE(\hat{Y}_i) \quad \text{if} \quad \rho < \frac{(\theta_{p1} + \theta_i) C_x}{2 C_y}; 1,2 \text{ (Yan and Tian (2010))} \tag{4.1}$$

5. NUMERICAL ILLUSTRATION

Data from Murthy [12] page 228 in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable) have been utilized for the purpose. The population parameters are given in Table 1 and Constants, Bias and MSE's of estimators are given in Table 2:

$$\% \text{ Relative Efficiency} = \frac{MSE(\text{Existing})}{MSE(\text{Proposed})} * 100 = 222.37\%$$

It is evident from the Table 2 that the proposed estimator has smallest bias as well as MSE value as compared to the already existing estimators. We also calculate the value of the condition given in (4.1) $\rho < \frac{(\theta_{p1} + \theta_i) C_x}{2 C_y}; 1,2$ i.e., **0.941 < 1.408**, which also is satisfied.

6. CONCLUSION

A new estimator has been proposed using the linear combination of Co-efficient of Kurtosis and Tri-mean of the auxiliary variable. Numerically it has also been proved that the estimator produces smallest bias as well as MSE value as compared to the already existing estimators. We conclude that the proposed estimator is proficient and more efficient than the existing estimators and be used in practical applications in estimating finite population mean.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:

The peer review history for this paper can be accessed here:
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