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Solution of Wind Speed Equation of Circulation Cyclone and Its Application

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

The wind speed equation of circular cyclone is a set of non-linear partial differential equations (PDE) with 4 unknown functions u (wind speed), ρ (density), p (pressure) and T (temperature) and is separated to 14 unknown functions, 4 unknown constants related by 15 equations. The solution is found analytically by method of separating variables, dimension analysis, and iteration method guided by periodic g-contractive mapping theorem. Isolated system is defined and the uniform circular motion (UCM) of steady flow in a horizontal plane is studied. Application to calculation of wind speed, temperature, mass density and mechanical energy are given. Comparing with results based on statistics, probability, or involving random factor, the result of this paper has 3 features: (1) it belongs to certainty-type; (2) four factors u_θ, ρ, p and T are linked; (3) these four factors are expressed by functions of r and z ; while in others, they belong to uncertain-type; three factors ρ, p and T are linked and are expressed by function of z only. Finally, conclusion is made.

Keywords: Wind speed equation; cyclone; partial differential equation; dimension analysis; steady flow; method of separating variables; iteration method; periodic g-contractive mapping theorem.

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1 Introduction

The obtained wind speed equation [1] is a set of non-linear partial differential equations (PDE) of four unknown functions u (wind speed), p (pressure), ρ (density) and T (temperature). Usually, a PDE with one unknown function appears in a differential operator, e.g., [2]. The case of **more unknown functions** in PDE are seldom seen in literature. If the unknown functions u, p, ρ and T are related by known relationship, then, the PDE can be changed to one unknown function of PDE and solved by usual methods. However, the relationship between these unknown functions are usually unknown before the PDE has been solved. Therefore adding complementary equation(s) becomes the key of solution. This case is similar to the case of "statically indeterminate problem" in mechanics, where the solution can not be obtained just by solving equilibrium equations, and complementary equation must be added. In this paper, complementary equations derived by method of iteration guided by the periodic g-contractive mapping theorem.

Fixed point theory is one of the active mathematical branch. It play an important role in proving the existence and getting solution of algebra equation, differential equation and integral equation, etc., by the powerful tool of iteration method.

A lot of researches on contractive mapping have been done [3]. Among these studies, the Banach contraction mapping theorem (or Banach fixed point theorem) [4] is the most famous, important and widely used theorem. However, the latter developed g-contractive mapping theorems [5-12] (originally proposed by Yun [5] and latter developed of applications in fields of mathematics, mechanics, prey-predator system, stock price, etc., [6 –12]) have improved and over the Banach fixed point theorem in two aspects: looser constraint and wider use for the latter. The Banach fixed point theorem needs the contractive ratio less than a constant lesser than 1. While, the g-contractive mapping theorem allows some of contractive ratios equal to or greater than 1, if the geometric mean of the contractive ratio is less than a constant lesser than 1. Further more, the g-contraction mapping theorem suits for periodic mapping with k-related fixed points, while the Banach fixed point theorem only suits for single mapping and get one fixed point. Here, the theorem is used to form complementary equations via method of iteration.

In section 2, the wind speed equation of a point in circular cyclone is separated into 14 unknown functions and 4 unknown constants related with 15 known functions. Three complementary equations are obtained by method of iteration guided by periodic g-contractive mapping theorem, and unknown constants are determined by known mass density and pressure data. By the use of separating variables, dimension analysis and complementary equations, the solution is finally obtained.

In section 3, isolated system is defined and a property is studied.

In section 4, application of the solution to calculation of wind speed, temperature, density of mass and mechanical energy of steady flow in UCM are given.

In section 5, comparing with results based on statistics, probability or involving random factor, the result of this paper has 3 features: (1) it belongs to certainty-type, (2) four factors u_θ, ρ, p and T are linked (3) they are expressed by function r and z; while in other methods, they belong to uncertain-type; three factors ρ, p and T are linked and are expressed only by function of z.

Finally, conclusion is made.

2 Solution of Wind Speed Equation of Circular Cyclone

At first, we summarize the wind speed equation of a point in UCM of cyclone. Which is obtained based on Boyle's law, Charles' law and Newton's law. It is a set of non-linear PDE with 4 unknown functions u_θ, ρ, p and T, independent variables r and z. Where (r, θ, z) denotes the position of the point in cylindrical coordinates.

2.1 Method of dimension analysis

The method of dimension analysis is a simplest method often used to check the correctness of an equation. The method states that the dimension of each term in both hand sides of an equation should be the same. In the following, the notation "$\dim A = B$" means that the dimension of A is B. The common unit system (m, s, N) is used as the unit of (length, time, force (N- Newton)) dimension system.

The 1 kg mass changes to force N, by $1kg\times 1m.s^{-2} = 1N$, or $1\ kg = 1\ N.m^{-1}s^2$.

The dimension of the 'combination of Boyle's law and Charles' law' i.e.,

$$pV = RT, \tag{2-1}$$

where p = pressure $(N.m^{-2})$, $V =$ volume (m^3), $R =$ constant $(N.m.degK^{-1})$, $T =$ temperature (K), $T = T_{^\circ C} + T_K \geq 0$, $T_{^\circ C}$= temperature of Celsius' degree, $T_K = 273.15℃$, the absolute temperature. The dimension of (2-1) is

$$\dim pV = N.cm = \dim RT.$$

The obtained component forms of wind speed equations of a point in UCM of steady flow of cyclone are (2-2) (relationship between u_θ and p) or (2-3) (relationship between u_θ and T) of [1]:

$$u_\theta^2(r,z) = g\left[\frac{\partial p(r,z)}{\partial z}\right]^{-1}[p(r,z) + r\frac{\partial p(r,z)}{\partial r}], \tag*{(2-2)$_A$}$$

$$\rho(r,z)g = \frac{\partial p(r,z)}{\partial z}, \tag*{(2-2)$_B$}$$

where $\rho = \rho(r,z) = m(r,z)/V(r.z)$ is the density of mass.

$$u_\theta^2(r,z) = g\left[\frac{\partial T(r,z)}{\partial z}\right]^{-1}\left[T(r,z) + r\frac{\partial T(r,z)}{\partial r}\right], \tag*{(2-3)$_A$}$$

$$mg = R\frac{\partial T}{\partial z}, \tag*{(2-3)$_B$}$$

Where $u_r = 0$, $u_\theta = u_\theta(r,z)$, and $u_z = gt$ (upward of u_z is confirmed by a recorded video [13]) are components of wind speed u; g is the acceleration of gravity; m is the mass of the point. $T = T(r,z)$ is the temperature; (r, θ, z) are cylindrical coordinates; t is the time. By integration of $g = \frac{\partial u_z}{\partial t} = \frac{\partial^2 s_z}{\partial t^2}$, we have $u_z(r,z,t) = gt$, $s_z = z = \frac{1}{2}gt^2$.

2.2 Method of separating of variables

Method of separating of variables is often used to solve PDE.

Suppose that all unknown functions can be separated. That is:

$$u_\theta^2 = u_\theta^2(r,z) = gB(r)C(z),\ (\dim u_\theta^2 = m^2s^{-2}, g = 9.81m.s^{-2}) \tag{2-4}$$

$$\rho(r,z) = \frac{m}{V} = bg^{-1}D(r)E(z), \quad (\dim \rho = kg.m^{-3} = N.m^{-4}.s^2, b = const.) \tag{2-5}$$

$$T(r,z)R = F(r)G(z), \quad (\dim TR = \dim pV = N.m^1) \tag{2-6}$$

$$p(r,z) = H(r)I(z), \quad (\dim p = N.m^{-2}) \tag{2-7}$$

Substituting (2-4), (2-6) into $(2\text{-}3)_A$, and separating variables, we have

$$C(z)\frac{\dot{G}(z)}{G(z)} = \frac{1}{B(r)}\left[1 + r\frac{F'(r)}{F(r)}\right] = k_1, \quad (k_1 = \text{const.}) \tag{2-8}$$

Where $F'(r) = dF(r)/dr$, $\dot{G} = dG(z)/dz$.

From (2-8), we have (2-9) and (2-10) :

$$F'(r) + F(r)[1 - k_1B(r)]/r = 0, \tag{2-9}$$

$$\frac{\dot{G}(z)}{G(z)} = \frac{k_1}{C(z)}, \tag{2-10}$$

Substituting (2-4), (2-7) into $(2\text{-}2)_A$, and separating variables, we have

$$C(z)\frac{\dot{I}(z)}{I(z)} = \frac{1}{B(r)}[1 + r\frac{H'(r)}{H(r)}] = k_2, \ (k_2 = \text{const}) \tag{2-11}$$

From (2-11), we have (2-12) and (2-13):

$$H'(r) + H(r)[1 - k_2B(r)]/r = 0, \tag{2-12}$$

$$\frac{\dot{I}(z)}{I(z)} = \frac{k_2}{C(z)}, \tag{2-13}$$

Substituting (2-5), (2-7) into $(2\text{-}2)_B$, and separating variables, we have

$$\frac{bD(r)}{H(r)} = \frac{\dot{I}(z)}{E(z)} = k_3, \quad (k_3 = \text{const.}) \tag{2-14}$$

From (2-14), we have (2-15) and (2-16):

$$D(r) = (\frac{k_3}{b})H(r), \tag{2-15}$$

$$\frac{dI(z)}{dz} = k_3E(z), \tag{2-16}$$

There are total 14 unknown functions $u_0^2, m, V, \rho, T, p, B, C, D, E, F, G, H, I$ and b, k_1, k_2, k_3 4 unknown constants, with (2-1), $(2\text{-}2)_A$, $(2\text{-}2)_B$, $(2\text{-}3)_A$, $(2\text{-}3)_B$, (2-4), (2-5), (2-6), (2-7), (2-9), (2-10), (2-12), (2-13), (2-15), (2-16) 15 known functions, we need 3 additional equations to solve 18 unknowns.

2.3 Complimentary equations derived by iteration method

2.3.1 Forming 8 related iteration series of $\{B_i\}, \{C_i\}, \{D_i\}, \{E_i\}, \{F_i\}, \{G_i\}\ \{H_i\}, \{I_i\}$

The 8 functions are related by 3 constants k_1, k_2, k_3 shown in (2-8), (2-11), and (2-14). Let us unify the notation of the 8 functions and mappings by

$$x_{i,j+1} = M_{i,j}x_{i,j}, \quad (i = 1,2, \dots, \ j = 1,2, \dots, 8) \tag{2-17}$$

$$x_{i,j} = (M_{i,j})^{-1}x_{i,j+1}, \tag{2-18}$$

Where $(M_{i,j})^{-1}$ is the inverse mapping of $M_{i,j}$, if it exists; subscript i denotes the number of iteration,

$$j = 1 = E_1(z); j = 2 = I_1(z); j = 3 = C_1(z); j = 4 = G_1(z); j = 5 = B_1(r); j = 6 = F_1(r); j = 7 = H_1(r); j = 8 = D_1(r). \quad (2\text{-}19)$$

Mapping $M_{i,j}: X \to X$, $(\forall x_j \in X = \text{Banach space})$, $d(x, y) = \text{distance for } x, y \in X$; $\text{norm}\|x\| = d(0, x)$.

2.3.2 Starting the iteration process

From [14], there are 5 layers in atmosphere, these layers are: Exosphere 700 – 10,000 km; Thermosphere 80 – 700 km; Mesosphere 50 – 80 km; Stratosphere 12 – 50 km; Troposphere 0 – 12 km. In this paper, only the troposphere is studied, where, in general, **air pressure and density decrease with increasing altitude**. The troposphere contains roughly **80% of mass** of Earth's atmosphere [15]. **Fifty percent of the total mass of atmosphere is located in the lower 5.6 km** of the troposphere.

From the above data, we have

$$\int_{z_0}^{12\text{km}} \text{Density}(z)\, dz = 0.8M_{\text{total}}, \quad (2\text{-}20)$$

$$\int_{z_0}^{5.6\text{km}} \text{Density}(z)\, dz = 0.5M_{\text{total}}, \quad (2\text{-}21)$$

Where $z_0 > 0$ is set to avoid mathematical infinity in calculation.

We start from $x_{1,1} = E_1(z)$.

$\text{Density}(z) = \rho(r, z) = \frac{b}{g} D_1(r)E_1(z)$, substituting ρ into (2-20)/(2-21), we have

$$\int_{z_0}^{12\text{km}} E_1(z)dz = 1.6 \int_{z_0}^{5.6\text{km}} E_1(z)dz, \quad (2\text{-}22)$$

The solution of (2-22) is not unique. The simplest one might be

$$x_{1,1} = E_1(z) = bz^{-1}, \quad (z > 0) \quad (2\text{-}23)_A$$

$$x_{1,1} = E_1(0) = z_0, \quad (z = 0) \quad (2\text{-}23)_B$$

$$\ln 12 \times 10^5 - \ln z_0 = 1.6[\ln 5.6 \times 10^5 - \ln z_0], \text{ or } z_0 = 1.2444, \text{ (m)} \quad (2\text{-}24)$$

Substituting $(2\text{-}23)_A$ into (2-16) and integrating, we have:

$$x_{1,2} = I_1(z) = M_{1,1}x_{1,1} = k_3 \int bz^{-1}\, dz = bk_3 \ln z, \ (z > 0) \quad (2\text{-}25)$$

$$I_1\dot{(z)} = bk_3z^{-1} = bk_3E_1(z) = bk_3x_{1,1}, \quad (2\text{-}26)$$

Substituting (2-25), (2-26) into (2-13), we have:

$$x_{1,3} = C_1(z) = k_2 \frac{I(z)}{I\dot{(z)}} = k_2 z \ln z = k_2 \frac{dz^{-1}}{z^{-1}}, \quad (2\text{-}27)$$

Substituting (2-27) into (2-10), we have:

$$\frac{dG_1(z)}{G_1(z)} = \frac{k_1}{k_2} \frac{d\ln z}{\ln z}, \quad (2\text{-}28)$$

Integrating both sides respect to z, we have

$$x_{1,4} = G_1(z) = \frac{k_1}{k_2} \ln z, \tag{2-29}$$

$x_{1,j}$ $j = 1 - 4$ have been found, however, there is no relation between function of r and function of z, except dimension relation. We need more boundary conditions to determine all unknowns.

From [16] the average atmospheric pressure at sea level is:

$$p(r, 0) = H_1(r)I_1(0) = 1 \text{ atm} = 1.013 \times 10^5 (N.m^{-2}). \tag{2-30}$$

Since the average atmospheric pressure is set to be independent of r, substituting $I_1(0) = k_3E_1(0) = k_3z_0$ into (2-30), we have:

$$\frac{H_1(r_1)k_3z_0}{b} = 1.013 \times 10^5 (N.m^{-2}) \tag{2-31}$$

By dimension analysis of (2-4), we have

$\dim u_\theta^2 = m^2s^{-2} = \dim[\, gB_1(r)C_1(z)] = m^1s^{-2}.m^1$,
Suppose that $B_1(r) = r$,

$$x_{1,5} = B_1(r) = \begin{cases} r, & r_0 \le r \le r_1 \\ r_0, & 0 \le r \le r_0 \end{cases}, \tag{2-32}$$

Where r_1 is the radius of the outer boundary of the circular cyclone; r_0 is the radius of inner boundary. The reason of why we consider two expresses of B(r) is that there is an **"eye"** in the center of cyclone. According to [17], r_1 and r_0 denote the radius of outer and inner boundary of the **"eye"** of the cyclone respectively. In [17], Hurricane Isabel (2003) as seen from Expedition 7 of the international space station, the "**eye"** is clearly visible in this view from space.

Substituting $B_1(r)$ into (2-9), we have:

$$F_1'(r) + F_1(r)[1 - k_1B_1(r)]/r = 0, \tag{2-33}$$

The solution of (2-33) is:

$$x_{1,6} = F_1(r) = F_1(r_0)\frac{r_0}{r}\exp[k_1(r - r_0)] \quad (r_0 \le r \le r_1) \tag*{(2-34)$_A$}$$

$$x_{1,6} = F_1(r) = F_1(r_0), \quad (0 \le r \le r_0) \tag*{(2-34)$_B$}$$

Substituting $B_1(r)$ into (2-12), we have:

$$H_1'(r) + H_1(r)[1 - k_2B_1(r)]/r = 0, \tag{2-35}$$

The solution of (2-35) is similar to (2-33), (note: k_2 instead of k_1) we have

$$x_{1,7} = H_1(r) = H_1(r_0)\frac{r_0}{r}\exp[k_2(r - r_0)], \ (r_0 \le r \le r_1) \tag*{(2-36)$_A$}$$

$$x_{1,7} = H_1(r) = H_1(r_0), \ (0 \le r \le r_0) \tag*{(2-36)$_B$}$$

Once $H_1(r)$ has been found, substituting $H_1(r)$ into (2-15), we have:

$$x_{1,8} = D_1(r) = D_1(r_0)\frac{H_1(r_1)k_3}{b}\frac{r_0}{r}\exp\left[k_2(r - r_0)\right], \quad (r_0 \le r \le r_1), \tag{2-37$_A$}$$

$$x_{1,8} = D_1(r) = D_1(r_0)\frac{H_1(r_1)k_3}{b}, \quad (0 \le r \le r_0) \tag{2-37$_B$}$$

From [16] the standard result of air mass density at 0℃, $p = 1\text{atm}$ (sea level) is set to be independent with r, let $r = r_1$, or $r = r_0$, we have the same value:

$$\rho(r_1, 0) = \frac{b}{g}D_1(r_1)E_1(0) = \frac{b}{g}D_1(r_1)z_0 = 1.29(\,N.m^{-4}.s^2), \tag{2-38}$$

Substituting (2-38) into (2-37)$_A$, and using (2-31), we have:

$$D_1(r_1) = D_1(r_0)\frac{H_1(r_1)k_3 z_0}{b}\frac{r_0}{r_1}\exp\left[k_2(r_1 - r_0)\right] = 1.29 \times 9.81\ (N.m^{-3}), \tag{2-39}$$

Substituting (2-30),（2-14）into (2-39), we have

$$k_2 = \frac{1}{r_1 - r_0}\ln\left[\frac{1.29\times 9.81}{1.013\times 10^5}\frac{r_1}{r_0}\right], \tag{2-40}$$

Where r_0 and r_1 are known from data (measured from the picture, e.g, [17]).

From (2-39), (2-31), and (2-37)$_A$, we have:

$$k_3 = \frac{1.29\times 10^5\times 9.81}{1.013\times 10^5}\frac{r_1}{r_0}\exp\left[k_2(r_1 - r_0)\right]^{-1}, \tag{2-41}$$

Now, all 8 unknown functions and unknown constants k_1, k_2, k_3 have been found in round 1 of iteration. We go to the second round, starting with $E_2(z)$ as round 1. Because the atmospheric data are only recorded with altitude z.

2.3.3 Iteration of the second round

From (2-8), (2-11) and (2-14), the second round iteration for $x_{2,1} = E_2$ and $x_{2,5} = B_2$ are calculated by inverse mapping, others are generated via $x_{2,1}$ and $x_{2,5}$.

$$x_{2,1} = (M_{1,1})^{-1}x_{1,2} = (M_{1,1})^{-1}©M_{1,1}x_{1,1} = P_{1,1}x_{1,1}, \tag{2-42}$$

Where f©g denotes the composition of f and g.

$$P_{1,1} = (M_{1,1})^{-1}©M_{1,1}, \tag{2-43}$$

$P_{1,1}$ is a **periodic mapping** with period k = 2.

$$x_{2,2} = M_{2,2}x_{2,1}, \quad x_{2,3} = M_{2,3}x_{2,1}, \quad x_{2,4} = M_{2,4}x_{2,1}, \tag{2-44}$$

We want to calculate $x_{2,5} = B_2(r)$, it can not be generated from other $x_{2,j}$, but only generated by inverse mapping. i.e.,

$$x_{2,5} = (M_{1,5})^{-1}©M_{1,5}x_{1,5} = P_{1,5}x_{1,5}, \tag{2-45}$$

$$x_{2,6} = M_{2,6}x_{2,5}, \quad x_{2,7} = M_{2,7}x_{2,5}, \quad x_{2,8} = M_{2,8}x_{2,7} \tag{2-46}$$

Now, the second round iteration is completed. If $B_{i+1} = P_i B_i = B_i = B^*$, then the series $\{B_i\}$ is called "**converges to fixed point** B^*". Thus, we have 8 fixed points,B^*, C^*, D^*,…, related each other by

$$H^* = M_{i,7}B^*,\ D^* = M_{i,8}H^*,\ F^* = M_{i,6}B^*,\ G^* = M_{i,3}C^*,\ I^* = M_{i,2}E^*. \tag{2-47}$$

2.3.4 The theorem of periodic g-contractive mapping -- conditions of convergent series $\{B_i\}, \{C_i\}, \{D_i\}, \{E_i\}, \{F_i\}, \{H_i\}, \{I_i\}$

Now, we consider the general case of iteration $i \to \infty$.

Definition 2.1. The ratio of

$\mathbf{r_i = \|x_{i+1} - x_i\| / \|x_i - x_{i-1}\|}$**, i≥1,** is called **contraction ratio** of x.

Here, we choice supper norm as the distance on $\|x_{i+1} - x_i\|$.

$\|x\| = \max|x(r,z)| = R_1 > 1$. ($R_1 < \infty$).

Definition 2.2 A sequential composite mapping $M_i : X \to X$ is called a **"g-contractive mapping"**, if for each $i \in N$, there exists a constant G, such that the geometric mean contraction ratio G_i satisfies:

$$0 \le G_i = (r_1 r_2 \dots r_i)^{1/i} < G < 1, \tag{2-48}$$

Definition 2.3 A sequential composite mapping $x_{i+1,j} = P_{i,j}x_{i,j}$ is called **"periodic g-contractive mapping with period k"**,

$$\text{if } M_{i,j-k} = M_{i,j}, \text{ where } P_{i,j} = M_{i,j} © M_{i,j-1} © \dots © M_{i,j-k}. \tag{2-49}$$

The theorem of periodic g-contractive mapping [5,10-12]:

Any periodic g-contractive mapping $P_{i,j}: X \to X$ in Banach space has a unique set of k related fixed points. (2-50)

2.3.5 Condition of stopping iteration

We set a threshold for stopping iterative calculation by b = const.

$$\|x_{i+1,j} - x_{i,j}\| \le b\|x_{1,j} - x_{0,j}\| = b.|x_{1,j} - 0|, (\forall j, b = \text{const. e.g.}, b = 1/2 < 1)$$

let G = b, then (2-48) holds, according to the theorem of periodic g-contractive mapping, it has a unique set of 8 related fixed point shown in (2-47)

Now, since the second round iteration is the same as the first round, due to no changing of data (2-20), (2-21) etc., thus $x_{2,1} = E_2 = E_1 = x_{1,1}$ and $x_{2,j} = x_{1,j}$ for all j. That means the convergence condition (2-48) has been satisfied. The 8 fixed points are found as shown in (2-47).

Summing up above results, we have:

$$u_\theta^2(r,z) = gB(r)C(z) = \begin{cases} g\,k_2 b\, r\, z \ln z, & (r_0 \le r \le r_1) \\ g k_2 b r_0 z \ln z, & (0 \le r \le r_0) \end{cases}, \tag{2-51}$$

$$\rho(r,z) = \frac{b}{g}D(r)E(z) = \begin{cases} D(r_0)\frac{k_3}{g}\frac{r_0}{r}\exp\{k_2 b(r-r_0)\}bz^{-1}, & (r_0 \le r \le r_1) \\ D(r_0)\frac{k_3}{g}bz_0, & (0 \le r \le r_0) \end{cases}, \tag{2-52}$$

$$RT(r,z) = F(r)G(z)$$
$$= \begin{cases} F(r_0)G(0)\frac{r_0}{r}\exp[k_1 b(r-r_0)]\frac{k_1}{k_2}\ln z, & (r_0 \le r \le r_1) \\ F(r_0)G(0)\frac{k_1}{k_2}\ln z, & (0 \le r \le r_0) \end{cases}, \tag{2-53}$$

$$p(r,z) = H(r)I(z) = \begin{cases} H(r_0)I(0)k_3\frac{r_0}{r}\exp[k_2 b(r-r_0)]\ln z, & (r_0 \le r \le r_1) \\ H(r_0)I(0)k_3 b\ln z, & (0 \le r \le r_0) \end{cases}, \tag{2-54}$$

3 Isolated System

An isolated system means the system has no exchanged or exchange in balanced with outside world. The phenomenon of isolated system often occurs in many fields, e.g., a system in an equilibrium state can be viewed as an isolated system. In atmosphere the isolated system often appears instantly, in seconds, hours or longer. However, the process of development of cyclone is a process of repeating the cycle of "equilibrium – non-equilibrium – (a new) equilibrium". So that the properties of isolated system should be studied.

3.1 Definition of boundary, entrance and exit between air groups

Definition 3.1 A region R with volume V wrapped by a zero-weighted membrane surface S_1,S_2 and S_3, if

$$u_N(a) = 0, \qquad \forall a \in S_1, \tag{3-1}$$

$$u_N(a) < 0, \qquad \forall a \in S_2, \tag{3-2}$$

$$u_N(a) > 0, \qquad \forall a \in S_3, \tag{3-3}$$

Then, S_1, S_2 and S_3 is called "**boundary", "entrance** ", and "**exit**" of R, respectively. Where N is the outer normal direction at point a on $S_i(i = 1,2,3)$ of R; $u_N(a)$ is the velocity component in N direction of point a. (3-1) means that there is no particle passing through S_1. (3-2) means that particles enter R through S_2. (3-3) means that particles exit R through S_3. Entrance or exit is also called **"front"**.

Definition 3.2 A region R defined as above, if $S_1 \neq 0$, $S_2 = S_3$,($\forall a \in S_2$, and also $\forall a \in S_3$), (i.e., a closed loop); or $\sum a_2 = \sum a_3$, $(a_2 \in S_2, a_3 \in S_3)$, (i.e., the number of particles moving in equals to that of moving out, or the mass m keeps unchanged), then, R is called an **"isolated region".**

Definition 3.3 An isolated system in a **stable equilibrium stat**e is herein defined by its mechanical energy reaches **minimum**.

The concept of stable equilibrium in mechanics is illustrated by a ball standing on the lowest point of a concave surface. Any disturbance let the ball moving from the equilibrium point will increase the energy. After the disturbance disappear, the ball tends to the equilibrium position.

3.2 Property. The mechanical energy of an UCM isolated system is minimum (comparing with non-UCM)

Example of isolate system: A ring tube with surfaces: $A_r = 2\pi r dz$, $A_{r+dr} = 2\pi(r+dr)dz$, $A_z = A_{z+dz} = 2\pi r dr$, volume $V = 2\pi r dr dz$, wrapped by zero-weighted membrane, fully filled points of UCM of steady flow in horizontal plane z is an isolated system. In which $u_r = 0$ on boundary surfaces A_r and A_{r+dr} shows that no particle passes through the boundary, and $S_2 = S_3$, therefore the ring tube is a closed loop, i.e., it is isolate. Its mechanical energy at time t is:

$$E = E_k + E_p = \int_0^V m\left[\frac{1}{2}u^2 + gz\right]dv = \int_0^V m\{\frac{1}{2}[u_\theta^2 + u_z^2] + gz\}dv, \quad (3\text{-}4)$$

Where E, E_k, E_p is the mechanical, kinetic and potential energy of the system respectively. $m = \rho(r,z)V$ is the mass.

Let the UCM system be disturbance by a small variation $\delta u_\theta\ (\to 0)$, then

$$\delta E = V\int_0^V \rho(r,z)u_\theta \delta u_\theta dv = 0, \quad (3\text{-}5)$$

$$\frac{\partial^2 E}{\partial u_\theta^2} = mV > 0, \quad (3\text{-}6)$$

(3-5), (3-6) show that E is at minimum. □

Any equilibrium state can be viewed as an isolated system, in which uniform velocity keeps minimum mechanical energy. Application examples, **uniform velocity suits for cruising speed, long distance running, etc. Any change of velocity to a non-UCM will increase mechanical energy of the system.**

The structure of a circular cyclone can be thought as that many closed loops of ring tube closely together forms a circular cyclone.

4 Application of the solutions (2-51) – (2-54)

The solutions (2-51) --（2-54）show the following results.

(1) The wind speed equation

$$u(r,z) = \sqrt{u_r^2 + u_\theta^2 + u_z^2} = \sqrt{g}\sqrt{bk_2 rz\ln z}, \quad (r_0 \le r \le r_1), \quad (4\text{-}1)_A$$

$$u(r,z) = \sqrt{g}\sqrt{bk_2 r_0 z\ln z}, \quad (0 \le r \le r_0), \quad (4\text{-}1)_B$$

$$\max u(r,z) = u(r_1,z) = \sqrt{gbk_2 r_1 z\ln z}, \quad (4\text{-}2)$$

$$\min u(r,z) = u(0,z) = \sqrt{gbk_2 r_0 z\ln z},, \quad (4\text{-}3)$$

(2) The tangent of the boundary funnel of the cyclone at $(\mathbf{r_1}, \mathbf{z})$.

$$u_\theta(r_1,z) = \sqrt{bk_2 g r_1 z\ln z}, (r_0 \le r \le r_1),\ u_z(r_1,z) = \sqrt{2gz}, (r_0 \le r \le r_1),$$

$$\tan\alpha = \frac{u_z(r_1,z)}{u_\theta(r_1,z)} = \sqrt{\frac{2}{bk_2 r_1 \ln z}}, \quad (r_0 \le r \le r_1), \quad (4\text{-}4)$$

Where α is the angle between horizontal plan and the tangential line of the funnel boundary at (r_1, z). If α can be measured, then b can be determined by (4-4).

(3) Mass density (2-52)

$\frac{\partial\rho}{\partial r} = 0$, we have $r = \frac{1}{bk_2}$, and

$$\max\rho(r,z) = \rho(1/bk_2, z) = D(r_0)\frac{k_3}{g}bk_2 r_0 \exp[-bk_2 r_0]z^{-1}, \quad (4\text{-}5)$$

(4) Temperature (2-53)

$\frac{\partial T}{\partial r} = 0$, we have $r = \frac{1}{bk_1}$, and

$$\max RT(r, z) = RT(1/bk_1, z) = F(r_0)G(0)bk_1r_0\exp\left[-bk_1r_0\right]\frac{k_1}{k_2}\ln z, \tag{4-6}$$

(5) Pressure (2-54)

$\frac{\partial p}{\partial r} = 0$, we have $r = \frac{1}{bk_2}$, and

$$\max p(r, z) = p(1/bk_2, z) = H(r_0)I(0)k_3bk_2r_0\exp\left[-bk_2r_0\right]z^{-1}, \tag{4-7}$$

(6) Calculation of mechanical energy of the UCM cyclone.

For a ring tube, substituting (2-51), (2-52) into (3-4), we have

$$E = 2\pi r\rho\left[\frac{1}{2}(u_\theta^2 + u_z^2) + gz\right]drdz$$

$$= \pi D(r_0)\frac{k_3}{g}r_0\exp[bk_2(r - r_0)]\,bz^{-1}\{[gbk_2rz\ln z + 2gz] + 2gz\}drdz$$

$$E_{total} = \pi D(r_0)k_3r_0b\{\int_{r_0}^{r_1} r\exp[bk_2(r - r_0)]\,dr + bk_2r_0\}\int_{z_0}^{z}[bk_2\ln z + 4]dz$$

$$= \pi D(r_0)r_0\frac{k_3}{k_2}\{\{r_1\exp[bk_2(r_1 - r_0)] - r_0\} - [\exp(bk_2(r_1 - r_0) - 1)\}\{bk_2[z\ln z - z_0\ln z_0] + 4[z - z_0]\}, \tag{4-8}$$

(4-8) is used to calculate the total mechanical energy of a UCM cyclone at time

5 Comparing

In the following, let A represent results of studies, method, calculation based on statistics, probability, or related to random factor; B represent the results of this paper.

(1). The base of A --- The ideal gas law:

$$pV = nRT, \tag{5-1}$$

(5-1) is the common form of the ideal gas law, firstly derived by Clapeyrong, E in 1834. Where p, V, T is the same as above, R is gas constant, equal to the product of Bolzmann constant and Avogadro constant; n is the number of moles. (5-1) relates with p, V, T and n. The element of the ideal gas law is mole or atom, it size belongs to micro-level. Application of ideal gas law to macro-problem often leads to results belonging to “uncertain-type”, since the random motion (Brown motion) is its basic assumption. While the base of B is the combination of Boyle’s law, Charles’ law, linked with Newton’s second law. It relates with u, p, V, T and ρ, no random factor has been involved. Therefore its result belongs to “certain-type”.

(2) Three factors p, V and T are related in A; while four factors u, p, V and T are related in B.

(3) Only altitude z are considered in calculation or data recorded in A; while $u(\rho, z), p(r, z), \rho(r, z)$ and $T(r, z)$ are considered in B.

6 Conclusion

(1) The method of separating variables works. The solution might be un-unique, since the constraints are looser (recorded data without r-th direction constraint).

(2) maximum wind speed locates on the edge of the cyclone r_1; minimum wind speed locates on the center of the cyclone. Staying at the center of the cyclone should be safe.

(3) $\rho(r, z), p(r, z)$ and $T(r, z)$ have similar distribution at the same time. Which shows that $\rho(r, z), p(r, z)$ and $T(r, z)$ are the functions of r and z in cyclone. This is a new result.

Competing Interests

Author has declared that no competing interests exist.

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