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ABSTRACT

Analogical reasoning is a complex process based on a comparison between two pairs of concepts or states of affairs (*aka*. the *source* and the *target*) for characterizing certain features from one to another. Arguments which employ this process to support their claims are called *analogical arguments*. Our goals are to study the structure and the computation for their defeasibility in light of the argumentation theory. Our proposed *assumption-based argumentation with predicate similarity* $\text{ABA}^{(p)}$ framework can be seen as an extension of assumption-based argumentation framework (ABA), in which not only assumptions can be used but also similarity of predicates is used to support a claim. $\text{ABA}^{(p)}$ labels each argument tree with an analogical degree and different ways to aggregate numerical values are studied toward gullible/skeptical characteristics in agent reasoning. The acceptability of analogical arguments is evaluated w.r.t. the semantics of abstract argumentation. Finally, we demonstrate that $\text{ABA}^{(p)}$ captures the argumentation scheme for argument from analogy and provides an explanation when it is used for persuasion.

KEYWORDS
analogical reasoning; metaphorical reasoning; description logic; assumption-based argumentation; persuasive reasoning

Introduction: Analogical Arguments and Their Acceptability

Analogical reasoning is a complex process based on a comparison between two pairs of concepts or states of affairs (*aka*. the *source* and the *target*) sharing some common features (Bartha 2010). This comparison is the ground of a specific type of inference called *argument from analogy*, in which the conclusion of an argument is attributed to a specific feature characterized from one to another (*cf.* the proposed models in (Copi, Cohen, and McMahon 2016; Davies 1988; Guarini, Butchart, Smith, and Moldovan 2009; Walton 2010; Walton, Reed, and Macagno 2008)). Despite the diversity, those models can be represented by the generic structure called

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an *argumentation scheme for argument from analogy* introduced in (Walton, Reed, and Macagno 2008) as follows:

Similarity Premise	Generally, case C_1 is similar to case C_2
Base Premise	A is true (false) in case C_1
Conclusion	A is true (false) in case C_2

This generic structure can be explained as follows. The similarity is regarded to hold between two cases. These cases could be two different 'concepts' or 'states of affairs'. Consequently, a property (*e.g.*, a feature A) attributes from one to another. Intuitively, this kind of structure can be represented as a logic program where A and C_i are appeared as the head and the body of an inference rule, respectively. Several attempts similar to this approach were developed in (Racharak et al. 2016, 2017; Raha, Hossain, and Ghosh 2008; Sun 1995).

A fundamental problem for this kind of reasoning is how to evaluate an analogical argument, *i.e.*, its acceptability. Basically, this problem amounts to investigations of the structure of analogical arguments and its defeasibility characteristics. At the abstract level, critical questions (CQ) (Walton, Reed, and Macagno 2008) associated with the argument scheme outlines several conditions of defeasibility:

CQ1 Is A true (false) in C_1?
CQ2 Are C_1 and C_2 similar in the respects cited?
CQ3 Are there important differences (dissimilarities) between C_1 and C_2
CQ4 Is there some other case C_3 that is also similar to C_1 except that A is false (true) in C_3?

These critical questions can be used to understand which analogical arguments should not be accepted. However, they do not address the following three basic problems: (1) how similarity/dissimilarity should be determined (which amounts to understand the notion of similarity); (2) how an analogical argument is constructed (which amounts to understand the structure of an analogical argument); and (3) how a conclusion drawn from the similarity premise and the base premise is warranted (which amounts to understand the evaluation of an analogical argument). The argumentation scheme and its critical questions do not involve these aspects concretely.

To address the first problem, we first take a look into the literature of similarity models. The most basic (but useful) one was developed by (Tversky 1977). In Tversky's model, an object is considered as a set of features. Then, the similarity of two objects is measured by the relationship between a number of common features and a number of different features. Nevertheless, not every feature need to be cited in analogical arguments, the studies in (Hesse 1965; Waller 2001; Weinreb 2016) reported that features used by the comparison should be 'relevant' to the attribution of the property. This leads to our study on characteristics of similarity models for analogical arguments in this work (*cf.* Section 3).

Addressing the second and the third problems involve in computing arguments in terms of argumentation with structure (or *structured argumentation*). It should be noted that argumentation (Dung 1995) is proven to be a promising platform to understand a non-monotonic and defeasible reasoning. With this viewpoint, these problems are indeed the problems of determining 'acceptable' analogical arguments w.r.t. argumentation semantics. That is, analogical arguments can attack (and be attacked by) other arguments. We show the correspondence between this attack–counterattack relationship and the defeasibility conditions of the argumentation scheme in this paper. More specifically, the definition of 'attack' is formally given in Section 4 and the link to the argumentation scheme is explained in Subsection 5.2.

This work uses the following dialogue as our running example. It is considered as analogical reasoning because **Agent$_1$** and **Agent$_2$** employ the perception of similarity as a means to justify their reasoning mechanism.

Agent$_1$: I think a goose can quack since it is like a duck.

Agent$_2$: No. Though it is like a duck, but to say that it can quack, we have to look into their vocal cords. Since they are built differently, it cannot quack.

There are several remarks which could be observed from the above example:

(1) Analogical reasoning is a kind of commonsense reasoning and defeasible reasoning. For example, **Agent$_1$** employs this kind of reasoning when he owns partial knowledge but a conclusion has to be drawn;
(2) This kind of reasoning can be used for 'persuasion', which conforms to the investigation in (Waller 2001). For example, **Agent$_1$** is trying to change the belief of **Agent$_2$** by arguing from the similarity of geese and ducks.
(3) Human beings are not certain about their conclusions of analogical reasoning. Their levels of certainty depend on the status of information, the interaction between arguments (*cf.* the counter-argument uttered by **Agent$_2$**), and types of agents *i.e.* gullible/skeptical agents. We further continue on this in Section 4.

In this paper, we focus on the computational aspect of analogical reasoning in argumentation, rather than the psychological modeling. Concretely, we study the structure of analogical arguments from the structured argumentation point of view, addressing the aforementioned problems. We analyze how the notion of 'concept similarity' contributes to the acceptability of analogical arguments. Section 2 reviews the basics of argumentation including assumption-based argumentation (ABA). Section 3 discusses a formal language for defining concepts and the notion of similarity measure of concepts. This notion equipped with ABA is used to define our proposed framework called *assumption-based argumentation with predicate similarity* ($\text{ABA}^{(\text{p})}$) in Section 4. This section also discusses about its relationship to different types of agents in analogical reasoning and Section 5 defines the notion of acceptability in argumentation and its link to Walton's scheme is explored. Finally, we relate our approach to others and discuss its future directions in Section 6 and Section 7, respectively.

Preliminary: Argumentation Framework and Its Structure

Abstract Argumentation

An abstract argumentation framework (AA) is a pair $(\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a set of *arguments* and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is called an *attack relation*. Arguments may attack each other; hence, it is clear that they may not stand together and their status are subject to an evaluation. Semantics for AA returns sets of arguments called *extensions*, which are *conflict-free* and *defend* themselves against attacks (Dung 1995).

Structured Argumentation

In AA, the structure and meaning of arguments and attacks are abstract. On the one hand, these characteristics enable the study of properties which are independent of any specific aspects (Baroni and Giacomin 2009). On the other hand, this generality features a limited expressivity and can be hardly adopted to model practical target situations. To fill out this gap, less abstract formalisms were considered, dealing in particular with the construction of arguments and the conditions for an argument to attack another *e.g.*, ASPIC^+ (Modgil and Prakken 2014), DeLP (Garca and Simari 2004), and assumption-based argumentation (ABA) (Dung, Kowalski, and Toni 2009). This work extends ABA and we include its basis here for self-containment.

Definition 2.1. An ABA framework is a quadruple $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ where

- $(\mathcal{L}, \mathcal{R})$ is a deductive system, in which $\mathcal{L}$ is a language and $\mathcal{R}$ is a set of inference rules,
- $\mathcal{A} \subseteq \mathcal{L}$ is a (non-empty) set, referred to as the set of *assumptions*,
- $^-$ is a total mapping from $\mathcal{A}$ to $\mathcal{L}$, where $\bar{\alpha}$ is the contrary of α.

We assume that the inference rules in $\mathcal{R}$ have the syntax $l_0 \leftarrow l_1, \ldots, l_n$ (for $n \geq 0$) where $l_i \in \mathcal{L}$. We refer to l_0 and $l_1, \ldots, l_n$ as the head and the body of the rule, respectively. We also represent the rule $l \leftarrow$ simply as l and restrict our attention to *flat* ABA framework (Bondarenko et al. 1997), *i.e.*, if $l \in \mathcal{A}$, then there exists no inference rules of the form $l \leftarrow l_1, \ldots, l_n \in \mathcal{R}$ for any $n \geq 0$.

As an example, the argumentation scheme for argument from analogy (*cf.* Section 1) can be represented in ABA as follows:[1]

$$hold(A, C_2) \leftarrow hold(A, C_1), sim(C_1, C_2), arguably(A, C_2)$$

where C_i represents different concepts or states of affairs, the conclusion $hold(A, C_2)$ may read "A holds in C_2"; also, the assumption premises $hold(A, C_1)$, $sim(C_1, C_2)$, and $arguably(A, C_2)$ may read "A holds in C_1", "C_1 and C_2 are similar to each other", and "the defeasible rule should not apply to the conclusion between A and C_2", respectively.

The above (domain-independent) inference rule is exemplified to the agent reasoning described in our running example. According to the biological family of birds, we know that ducks and geese are belonged to the same family *i.e.* 'Anatidae'. These birds are adapted for swimming, floating on the water surface, *etc*. Though they are under the same family, ducks and geese are different. This information supports us to conclude that ducks and geese are similar. We represent the assumptions as follows.

$$hold(quack, duck); \quad sim(duck, goose)$$

where the assumptions $hold(quack, duck)$ and $sim(duck, goose)$ states that "ducks can quack" and "ducks and geese are similar to each other", respectively.

Given an ABA framework, an argument in favor of a sentence $c \in \mathcal{L}$ supported by a set S of assumptions, denoted by $S \vdash c$, is a backward deduction from c to S obtained by applying backward the rules in $\mathcal{R}$, *e.g.* $\{hold(quack, duck), sim(duck, goose), arguably(quack, goose)\} \vdash hold(quack, goose)$.

In ABA, the notion of attack between arguments is defined in terms of the contrary of assumptions, *i.e.*, an argument $S_1 \vdash c_1$ attacks another (or the same) argument $S_2 \vdash c_2$ iff c_1 is the contrary of an assumption in S_2.

In general, the contrary of an assumption is a sentence representing a challenge against the assumption and can be suggested by critical questions (CQ) of an argumentation scheme (*cf.* page 2 for its description). For instance, the assumption $hold(A, C_1)$ can be challenged by providing a negative answer to CQ1 *i.e.* $\neg\, hold(A, C_1)$, where symbol $\neg$ denotes the classical negation. Supplying a negative answer to CQ2 and CQ3 can also be understood as proving the contrary $\neg\, sim(C_1, C_2)$ (*i.e.* C_1 and C_2 are dissimilar to each other) of the assumption $sim(C_1, C_2)$. A negative answer to CQ4 can be understood as showing the contrary $\neg\, hold(A, C_2)$ of the assumption

$$
\begin{array}{ll}
\mathcal{R}: & hold(A, C_2) \leftarrow hold(A, C_1), sim(C_1, C_2), arguably(A, C_2); \\
& \neg hold(A, C_2) \leftarrow sim(C_1, C_2), sim(C_1, C_3), hold(A, C_1), \neg hold(A, C_3); \\
& \neg hold(quack, C) \leftarrow cord(A, C), \neg built(quack, A); \\
& cord(cord_g, goose); \qquad \neg built(quack, cord_g) \\
\mathcal{A}: & hold(quack, duck); \qquad sim(duck, goose); \qquad arguably(quack, goose) \\
\overline{}: & \overline{hold(quack, duck)} = \neg hold(quack, duck); \\
& \overline{sim(duck, goose)} = \neg sim(duck, goose); \\
& \overline{arguably(quack, goose)} = \neg hold(quack, goose)
\end{array}
$$

Figure 1. ABA framework for the running example.

arguably(A, C_2). This contrary $\neg$ *hold*(A, C_2) may be defined by an additional (domain-independent) inference rule: $\neg$ *hold*$(A, C_2) \leftarrow$ *sim*(C_1, C_2), *sim*(C_1, C_3), *hold*(A, C_1), $\neg$ *hold*(A, C_3). Contraries may also be derived via a chain of rules, *e.g.* $\neg hold(quack, C) \leftarrow cord(A, C), \neg built(quack, A)$; $cord(cord_g, goose)$; $\neg built(quack, cord_g)$, representing an abnormality condition that their vocal cords are built differently. The overall ABA framework is summarized in Figure 1.

A Formal Notion of Concepts and Similarity

Subsection 2.2 shows that the argumentation scheme for argument from analogy may be encoded into an ABA. However, several general problems still remain uncleared, for instance, how the similarity predicate *sim*(C_1, C_2) should be supplied to an ABA framework? We note that this point has already been mentioned in Section 1.

The example 'biological family of birds' apparently illustrates that similarity of concepts (or states of affairs) can be considered from their descriptions or their taxonomy *e.g.* "ducks are a kind of birds which are adapted for swimming". Regarding this observation, any frameworks which encode 'argument from analogy' must provide mechanisms to formalize the description of concepts. In a very simple way, we may formalize the description of concepts in terms of inference rules. For instance, $duck(X) \leftarrow waterbird(X), feature\_d(X)$ where $feature\_d(X)$ represents a unique characteristic for ducks. When inference rules are grounded, ones can employ the model theory to derive the similarity between predicates as in (Goebel 1989).

Though using inference rules can encode our example, other knowledge representation formalisms which provide more expressivity may be also used to encode concepts *e.g.* description logic (DL) (Baader et al. 2007) or (other fragments of) first-order logic. For example, the same description can be formalized based on DL as: Duck $\sqsubseteq$ WaterBird, in which '$\sqsubseteq$' is read as 'is a'. Successful examples of DL knowledge bases are ontologies in medicine and bioinformatics *e.g.* Snomed ct (www.snomed.org) or Go (www.geneontology.org).

The above investigation suggests that any ABA framework extended for analogical reasoning should supply with a module containing the formalized descriptions of concepts and the logical relationship between them. This section concentrates on concepts formalized using DL formalism and a similarity measure is defined for those concepts.

Preference Context for Having Relevance

Similarity of concepts is oftentimes context-sensitive and can be recognized from the comparison of features shared between them. Nevertheless, (Hesse 1965; Waller 2001; Weinreb 2016) reported that features used in comparisons should be 'relevant' to the attribution of the property. This means that there must be ways of expressing aspects of a context in consideration. In the following, we introduce a notion called *preference context* which can be used to express a considering context in DL formalism.

In general, DL concept descriptions C, D (or simply concepts) can be defined inductively through a set CN of concept names and a set RN of role names as: $C, D ::= A \mid \top \mid C \sqcap D \mid \exists r.C \mid \forall r.C$ where $A \in \mathsf{CN}$, $r \in \mathsf{RN}$, $\top$ denotes the *top concept*, and $\sqcap, \exists, \forall$ are called *concept constructors*. A terminological knowledge base or TBox $\mathcal{T}$ is a set of formulae defined over concepts. Examples of TBox formulae are $C \sqsubseteq D$ (denoting "concept C is a kind of concept D") and $C \equiv D$ (denoting "concept C is definitely concept D"). The following definition defines different ways of preferences expressed over DL concepts.

Definition 3.1. Let I_1, I_2 be non-empty sets equipped with partial orders $\leq_{I_1}$ and $\leq_{I_2}$, respectively; for any $x \in I_1$, for any $y \in I_2$, it holds that $x \leq y$; and a special element $n \notin I_1 \cup I_2$ representing the neutral. Let S, D be a non-empty sets equipped with partial orders $\leq_S$ and $\leq_D$, respectively. A preference context (denoted by $\mathfrak{p}$) is a quintuple $\langle \mathsf{ic}, \mathsf{ir}, \mathsf{sc}, \mathsf{sr}, \mathsf{d} \rangle$ where $\mathsf{ic}, \mathsf{ir}, \mathsf{sc}, \mathsf{sr}, \mathsf{d}$ are 'partial' functions such that:

- $\mathsf{ic} : \mathsf{CN} \to I_1 \cup \{n\} \cup I_2$ captures the importance of concept names;
- $\mathsf{ir} : \mathsf{RN} \to I_1 \cup \{n\} \cup I_2$ captures the importance of role names;
- $\mathsf{sc} : \mathsf{CN} \times \mathsf{CN} \to S$ captures the similarity of concept names;
- $\mathsf{sr} : \mathsf{RN} \times \mathsf{RN} \to S$ captures the similarity of role names; and
- $\mathsf{d} : \mathsf{RN} \to D$ captures the importance factor of a quantified role (*e.g.* $\exists r$) in relation to the corresponding concept (*e.g.* C) for quantified concepts (*e.g.* $\exists r.C$).

Now, we exemplify the above functions. Let $I_2 := \{i_1, i_2\}$ where $i_1 \leq_{I_2} i_2$. Saying that an occurrence of Bird is more important than that of Lizard in a description can be expressed as $\mathsf{ic}(\mathsf{Bird}) = i_2$ and $\mathsf{ic}(\mathsf{Lizard}) = i_1$. Other functions are also straightforward to understand except d. Hence, we merely illustrate it next. Let $D := \{d_1, d_2\}$ where $d_1 \leq_D d_2$. Suppose that ones would like to compare between $\exists\mathsf{float}.\mathsf{Water}$ and $\exists\mathsf{float}.\mathsf{Air}$ with a consideration that being 'floatable' is more influential than other properties. Then, we may express as $\mathsf{d}(\mathsf{float}) = d_2$ and other role names are mapped to d_1.

A well-investigated concrete notion of preference context is the *preference profile* (denoted by π) introduced in (Racharak, Suntisrivaraporn, and Tojo 2018) where $I_1 := [0, 1)$, $n := 1$, $I_2 := (1, 2]$, $S := [0, 1]$, and $D := [0, 1]$. Next, we discuss that preference context can be considered in the development of concept similarity measures.

Concept Similarity under Preferences

This subsection defines a generic notion of concept similarity measure from two main observations. First, similarity of concepts should be 'subjective to' a preference context. This suggests that any similar measures for concepts should supply with tunable parameters w.r.t. the preference context. Second, similarity of concepts is a 'direct generalization' of equality relation for concepts (or the *concept equivalence* relation). In DL, two concept descriptions C, D are 'equivalent' w.r.t. TBox $\mathcal{T}$ (in symbols, $C \equiv_{\mathcal{T}} D$) iff their semantic representations $C^{\mathcal{I}}, D^{\mathcal{I}}$ are the same, *i.e.* $C^{\mathcal{I}} = D^{\mathcal{I}}$, for every model[2] $\mathcal{I}$ of $\mathcal{T}$. We adopt these two viewpoints and introduce the following.

Definition 3.2. Let $\mathfrak{P}$ be an infinite set of preference contexts where $\mathfrak{p} \in \mathfrak{P}$, $\mathsf{Con}(\mathsf{CN}, \mathsf{RN})$ be a set of concept descriptions constructed from CN and RN where $C, D \in \mathsf{Con}(\mathsf{CN}, \mathsf{RN})$, and $\mathcal{T}$ be a TBox. Then, a *concept similarity under preferences* is a family of functions $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}} : \mathsf{Con}(\mathsf{CN}, \mathsf{RN}) \times \mathsf{Con}(\mathsf{CN}, \mathsf{RN}) \to [0, 1]$ such that

$$\forall \mathfrak{p}' \in: \mathfrak{P}\; C \overset{\mathfrak{p}'}{\sim}_{\mathcal{T}} D = 1 \Leftrightarrow C \equiv_{\mathcal{T}} D$$

(called *preference invariance w.r.t. concept equivalence*) holds; and

- $C \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} D = 1$ indicates *maximal similarity* (or concept equivalence) under preference context $\mathfrak{p}$ w.r.t. $\mathcal{T}$ between concept descriptions C and D,
- $C \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} D = 0$ indicates *having no relation* under preference context $\mathfrak{p}$ w.r.t. $\mathcal{T}$ between concept descriptions C and D.

The reason we require preference invariance w.r.t. concept equivalence because we do not want to allow the usage of any preference context to effect on the perception of semantically identical concept descriptions.

There also exist well-developed functions for concept similarity under preferences such as the function sim^{π} for an unfoldable TBox introduced in (Racharak, Suntisrivaraporn, and Tojo 2018). Basically, this function computes the degree of similarity between concepts (*e.g.* Duck and Goose) by rather calculating from their corresponding description trees (*e.g.* $\mathcal{T}_{\mathsf{Duck}}$ and $\mathcal{T}_{\mathsf{Goose}}$, respectively). Since sim^{π} can be considered as an instance of $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}}$, this function enables the agent to express his preferences in terms of preference profile such that the degree of similarity between two concept descriptions is identified w.r.t. his perception. The following example shows that this similarity measure can be used to provide a numerical value representing the degree of similarity perception, in which the function hd^{π} computes the degree of directional tree similarity w.r.t. preference profile π. Their definitions are omitted to show here due to the limited space.

Example 3.3. Let TBox $\mathcal{T} := \{\mathsf{Duck} \sqsubseteq \mathsf{WaterBird}, \mathsf{Goose} \sqsubseteq \mathsf{WaterBird}\}$ and the default preference profile π_0 (also, introduced in (Racharak, Suntisrivaraporn, and Tojo 2018)) represents the agent's preferences in the default manner *i.e.* preferences are not given.

We compute the similarity of Duck and Goose using sim^{π} with the preference profile π_0 *i.e.* $\mathsf{sim}^{\pi_0}(\mathsf{Duck}, \mathsf{Goose}) = (\mathsf{hd}^{\pi_0}(\mathcal{T}_{\mathsf{Duck}}, \mathcal{T}_{\mathsf{Goose}}) + \mathsf{hd}^{\pi_0}(\mathcal{T}_{\mathsf{Goose}}, \mathcal{T}_{\mathsf{Duck}}))/2$, where $\mathcal{T}_{\mathsf{Duck}}, \mathcal{T}_{\mathsf{Goose}}$ represents the concept trees of Duck and Goose, respectively. Since $\mathsf{hd}^{\pi_0}(\mathcal{T}_{\mathsf{Duck}}, \mathcal{T}_{\mathsf{Goose}}) = (1)[(1 \cdot \max\{1, 0\} + 1 \cdot \max\{0, 0\})/(1 + 1)] = 1/2$ and $\mathsf{hd}^{\pi_0}(\mathcal{T}_{\mathsf{Goose}}, \mathcal{T}_{\mathsf{Duck}}) = 1/2$. Then, $\mathsf{sim}^{\pi_0}(\mathsf{Duck}, \mathsf{Goose}) = 1/2$. This number indicates the degree of similarity between Duck and Goose in the normal perception.

Assumption-based Argumentation with Predicate Similarity

We have discussed the theoretical analysis of using ABA framework to model the argumentation scheme for argument from analogy and concept similarity under preferences for understanding the degree of similarity between concepts in Subsection 2.2 and Section 3, respectively. Though using ABA alone could model the argumentation scheme for argument from analogy, it came up with several difficulties as follows.

First, ABA does not concretely describe where the source of similarity premises comes from, how a notion of concept similarity should be involved, how 'relevance' of concept similarity is defined and effects the degree of analogical arguments, and how analogical arguments should interact with normal arguments in case of persuasion. These problems are basically related to redefining both the notion of structured arguments and the framework in a way that arguments' types can be classified.

Second, an analogical argument should be associated with a particular degree since each analogy used to support a claim is associated with a unit interval [0, 1]. This degree should also contribute to the attack relation between arguments. It is worth mentioning that similarity could be 'qualitative' in a sense that ones may only perceive if two concepts are similar or not. In this case, a certain threshold should be defined for being similar and each analogical argument could be associated with a binary $\{0, 1\}$ where 1 indicates 'similar' and 0 indicates 'not similar'.

Third, different rational agents may value arguments supported by analogies unequally, depending on their characteristics. This point is related to different styles of making judgment. For example, there could be a 'gullible' agent who always gives a high degree on every analogical argument; or a 'skeptical' agent vice versa.

To address the first difficulty, we extend the original ABA framework to *assumption-based argumentation with predicate similarity* (denoted by $\mathrm{ABA}^{(\mathrm{p})}$) by identifying necessary components to form analogical arguments. In the following, the extended framework considers any arbitrary description language although DL terminological formalism is used in our running example.

Definition 4.1. An $\mathrm{ABA}^{(\mathrm{p})}$ is a 10-tuple $\langle \mathcal{L}_D, \mathcal{R}, \mathcal{A}, \overline{}, \mathcal{L}_T, \mathcal{T}, ^{\mathcal{M}}, \overset{\mathfrak{p}}{\sim}_{\mathcal{T}}, \mathfrak{p}, \mathcal{F} \rangle$ where $(\mathcal{L}_T, \mathcal{T})$ is a module formalizing descriptions of concepts with a language $\mathcal{L}_T$ and a set $\mathcal{T}$ of formulae (constructed from $\mathcal{L}_T$) representing definitions of concepts, $^{\mathcal{M}}$ is a partial mapping from the predicate of sentences in $\mathcal{L}_D$ to concepts in $\mathcal{L}_T$, $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}} : \mathcal{L}_T \times \mathcal{L}_T \to [0, 1]$ is a certain concept similarity w.r.t. $\mathcal{T}$ under preference context $\mathfrak{p}$, $\mathcal{F}$ is an annotation function for each entire argument to a numerical value[3], $\overline{}$ is a total function mapping from $\mathcal{A} \cup \mathcal{AN}$ to $\mathcal{L}_D$, where $\mathcal{AN} := \{P \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q \mid P^{\mathcal{M}} \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q^{\mathcal{M}} \in (0, 1],$ for any $P(t_1, \ldots, t_p),$ $Q(t_1, \ldots, t_p) \in \mathcal{L}_D\}$[4] representing a set of analogies, and $\mathcal{L}_D, \mathcal{R}, \mathcal{A}$ are as defined in ABA framework. An argument for $c \in \mathcal{L}_D$ (the conclusion or claim) supported by $\mathcal{S} \subseteq \mathcal{A} \cup \mathcal{AN}$, is a tree with nodes labeled by sentences in $\mathcal{L}_D \cup \mathcal{AN}$, by sentences of the special form $?(\varphi, \psi, \varsigma)$ representing a defeasible condition of sentence φ concluded from an analogy between ψ and ς, or by the special symbol $\square$ representing an empty set of premises, such that:

- the root is labeled by c;
- for every node N,

∘ if N is a leaf, then N is labeled by an assumption in $\mathcal{A} \cup \mathcal{AN}$, an assumption of the form $?(\varphi, \psi, \varsigma)$, or by $\square$,
∘ if N is not a leaf, l_N is the label of N, and there is an inference rule $l_N \leftarrow b_1, \ldots, b_m$ $(m \geq 0)$ in $\mathcal{R}$, then
either $m = 0$ and the child of N is $\square$
or $m > 0$ and N has m children, labeled by $b_1, \ldots, b_m$, respectively,
∘ if N is not a leaf, l_N is the label of N where $l_N := P(t_1, \ldots, t_p)$, there is an analogy $P \overset{\mathfrak{p}}{\sim}_\tau Q$ in $\mathcal{AN}$, and there is
either an inference rule $Q(t_1, \ldots, t_p) \leftarrow b_1, \ldots, b_m$ $(m \geq 0)$ in $\mathcal{R}$
or $Q(t_1, \ldots, t_p)$ in $\mathcal{A}$, then
N has 3 children, labeled by $P \overset{\mathfrak{p}}{\sim}_\tau Q$, $?(l_N, P, Q)$, $Q(t_1, \ldots, t_p)$;

- $\mathcal{S}$ is the set of all assumptions labeling the leaves.

$(\mathcal{L}_T, \mathcal{T})$ can be defined for any kinds of terminological formalism specified by means of a language $\mathcal{L}_T$ and a set of formulae $\mathcal{T}$. For example, a DL terminological knowledge base can be recast as $\mathcal{L}_T := \mathsf{CN} \cup \mathsf{RN}$ and $\mathcal{T}$ is a TBox constructed from $\mathcal{L}_T$.

We note that $?(\varphi, \psi, \varsigma)$ can be read as "conclusion φ supported by an analogy between ψ and ς is opened for challenging". A challenge of φ could be the contrary of φ, which may be possibly drawn from other analogies (*aka*. counter-analogies) or chains of inference rules. For example, a challenge of "*sound*$_2$ created by *bird*$_2$ is duck's sound" is an evidence that *sound*$_2$ is *honk* sound. Like ABA, assumptions are the only defeasible component in ABA$^{(\mathfrak{p})}$ and they are used to support a conclusion. For the sake of simplicity, we clearly separate analogical assumptions from standard assumptions. That is, an argument for c supported by standard assumption $\mathcal{S}^{\mathcal{A}} \subseteq \mathcal{A}$ and analogical assumption $\mathcal{S}^{\mathcal{AN}} := \mathcal{S} \setminus \mathcal{S}^{\mathcal{A}}$ is denoted by $\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c$ (*i.e.* $\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} = \mathcal{S}$ such that $\mathcal{S}^{\mathcal{A}} \cap \mathcal{S}^{\mathcal{AN}} = \emptyset$). When $\mathcal{S}^{\mathcal{AN}}$ is empty *i.e.* $\mathcal{S}^{\mathcal{A}} \cup \emptyset \vdash c$, we call such an argument a *standard argument*. Otherwise, we call it an *analogical argument*. This style of writing helps recognizing analogical arguments and standard arguments at first glance.

It is worth noting that the study of analogical reasoning in logical systems is not new since several studies do exist. For example, Goebel (1989) provided a form of analogical reasoning in terms of a system of hypothetical reasoning, Sun (1995) integrated rule-based and similarity-based reasoning in a connectionist model. In argumentation systems, Racharak et al. (2016) studied an implementation of analogical reasoning using an argument-based logic programming and (Racharak et al. 2017) proposed an idea to combine answer set programming with description logic. This work makes a continuous study of these papers by generalizing (Racharak et al. 2017) to ABA.

To address the second difficulty, we define the function $f : \mathcal{S} \to [0, 1]$ for annotating (both standard and analogical) assumptions as follows:

Definition 4.2. Given a set $\mathcal{S}$ of assumptions, a partial mapping $^{\mathcal{M}}$ from the predicate of sentences in $\mathcal{L}_D$ to concepts in $\mathcal{L}_T$, and $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}} : \mathcal{L}_T \times \mathcal{L}_T \to [0,1]$ is a certain concept similarity w.r.t. terminological formalism $\mathcal{T}$ under preference context $\mathfrak{p}$, the (total) annotation function $f : \mathcal{S} \to [0,1]$ is defined, for any $a \in \mathcal{S}$, as:

$$f(a) = \begin{cases} P^{\mathcal{M}} \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q^{\mathcal{M}} & \text{if } a \text{ is of the form } P \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q \\ P^{\mathcal{M}} \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q^{\mathcal{M}} & \text{if } a \text{ is of the form } ?\,(l_N, P, Q) \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Intuitively, standard assumptions are labeled with 1 to correspond with the fact that similarity relation is bound by 1 (we note that 1 is used in $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}}$ to indicate the maximal similarity). Next, we extend f to the function $\mathcal{F}$ for annotating arguments. Each annotation represents the degree of each entire argument.

Definition 4.3. Let $\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c$ be an argument. Then, a function $\mathcal{F}$ for annotating an entire argument is defined as:

$$\mathcal{F}(\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c) = \begin{cases} \otimes\{f(a_i), f(an_j)\} & \text{if } \mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \neq \emptyset \\ 1 & \text{otherwise} \end{cases} \tag{2}$$

where $a_i \in \mathcal{S}^{\mathcal{A}}$, $an_j \in \mathcal{S}^{\mathcal{AN}}$, and $\otimes$ is a triangular norm (t-norm).

Since the above definition employs the notion of t-norm, we include its basis here for self-containment. A function $\otimes : [0,1]^2 \to [0,1]$ is called a *t-norm* iff it fulfills the following properties for all $x, y, z, w \in [0,1]$: (1) $x \otimes y = y \otimes x$ (commutativity); (2) $x \leq z$ and $y \leq w \Rightarrow x \otimes y \leq z \otimes w$ (monotonicity); (3) $(x \otimes y) \otimes z = x \otimes (y \otimes z)$ (associativity); (4) $x \otimes 1 = x$ (identity). A t-norm is called *bounded* iff $x \otimes y = 0 \Rightarrow x = 0$ or $y = 0$. There are several reasons for the use of a t-norm. Firstly, it is the generalization of the conjunction in propositional logic. Secondly, the operator *min* (*i.e.* $x \otimes y = \min\{x, y\}$) is an instance of a bounded t-norm. This reflects an intuition that the strength of an argument depends on the used 'weakest' analogical assumptions. Lastly, 1 acts as the neutral element for t-norms.

Concerning the third difficulty, the choice of $\otimes$ (*cf.* Table 1 for its examples) can represent a type of a rational agent in analogical reasoning. For example, a gullible/skeptical agent may give a high/low degree to his answer when his answer is derived from analogies. We formalize this characteristic as follows.[5]

Table 1. Some instances of the operator $\otimes$.

Name	Notation	$x_1 \otimes x_2 =$
Minimum	$\otimes_{\min}$	$\min\{x_1, x_2\}$
Product	$\otimes_{\text{mlt}}$	$x_1 \cdot x_2$
Hamacher product	$\otimes_{H_0}$	0 if $x_1 = x_2$; otherwise $\frac{x_1 \cdot x_2}{x_1 + x_2 - x_1 \cdot x_2}$

Definition 4.4. Let $\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c$ be an argument; also, $\mathcal{F}_1$ and $\mathcal{F}_2$ be two different functions representing different agents. Then, $\mathcal{F}_1$ is *more gullible* than $\mathcal{F}_2$ if $\mathcal{F}_1(\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c) \geq \mathcal{F}_2(\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c)$. On the other hand, $\mathcal{F}_1$ is *more skeptical* than $\mathcal{F}_2$ if $\mathcal{F}_1(\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c) \leq \mathcal{F}_2(\mathcal{S}^{\mathcal{A}} \cup \mathcal{S}^{\mathcal{AN}} \vdash c)$. Lastly, $\mathcal{F}_1$ and $\mathcal{F}_2$ are *identical* if $\mathcal{F}_1$ are both gullible and skeptical to $\mathcal{F}_2$.

The following theorem is an aid to help deciding which operator $\otimes$ should be chosen for $\mathcal{F}$ in $\text{ABA}^{(\text{p})}$. That is, if an agent strongly recognizes analogical principles, we may choose the most gullible function (*i.e.* $\otimes_{\min}$). On the other hand, we may choose the skeptical function (*i.e.* $\otimes_{\text{mlt}}$) if an agent weakly recognizes analogical principles.

Theorem 4.5. *From Table 1 and let* $x_1, x_2 \in (0, 1]$. *Then,* $\otimes_{\text{mlt}} \leq \otimes_{H_0} \leq \otimes_{\min}$.

Proof. *(Sketch) We show the following inequality*:

$$x_1 \cdot x_2 \leq \frac{x_1 \cdot x_2}{x_1 + x_2 - x_1 \cdot x_2} \leq \min\{x_1, x_2\}$$

That is, we show $x_1 \cdot x_2 \leq \frac{x_1 \cdot x_2}{x_1 + x_2 - x_1 \cdot x_2}$ *as follows:*

$$x_1 \cdot x_2 \leq \frac{x_1 \cdot x_2}{x_1 + x_2 - x_1 \cdot x_2} \Leftrightarrow 1 \leq \frac{1}{x_1 + x_2 - x_1 \cdot x_2} \Leftrightarrow x_1 + x_2 - x_1 \cdot x_2 \leq 1$$
$$\Leftrightarrow x_2 - x_1 \cdot x_2 \leq 1 - x_1 \Leftrightarrow (1 - x_1) \cdot x_2 \leq 1 - x_1 \Leftrightarrow x_2 \leq 1 (\text{by assumption})$$

Lastly, we show $\frac{x_1 \cdot x_2}{x_1 + x_2 - x_1 \cdot x_2} \leq \min\{x_1, x_2\}$ *in the similar fashion.* □

Attacks in ABA are defined in terms of the contrary of assumptions (*cf.* Subsection 2.2). However, argument trees and their supporting assumptions in $\text{ABA}^{(\text{p})}$ are labeled with numbers. This is clear that the current definition of attacks in ABA is not appropriate for handling attacks in $\text{ABA}^{(\text{p})}$. To define the notion of attacks in $\text{ABA}^{(\text{p})}$, we extend the original definition of attacks in ABA to take into account the numbers. In addition, the extended definition imposes a particular restriction on the usage of analogical reasoning for 'persuasion' *i.e.* analogical arguments are always preferable to standard arguments. These characteristics are formally defined as follows.

Definition 4.6. Let function $\overline{}$, function $\mathcal{F}$, and function f be as defined in Definition 4.1, Definition 4.2, and Definition 4.3, respectively. An argument $\mathcal{S}_1^{\mathcal{A}} \cup \mathcal{S}_1^{\mathcal{AN}} \vdash c_1$ *attacks* an argument $\mathcal{S}_2^{\mathcal{A}} \cup \mathcal{S}_2^{\mathcal{AN}} \vdash c_2$ iff the following satisfies:

- If $\mathcal{S}_1^{\mathcal{AN}} \neq \emptyset$ and $\mathcal{S}_2^{\mathcal{AN}} = \emptyset$, then c_1 is the contrary of an assumption in $\mathcal{S}_2^{\mathcal{A}}$;
- Otherwise, c_1 is the contrary of an assumption in $\mathcal{S}_2^{\mathcal{A}} \cup \mathcal{S}_2^{\mathcal{AN}}$ (*i.e.* $x \in \mathcal{S}_2^{\mathcal{A}} \cup \mathcal{S}_2^{\mathcal{AN}}$ and $\overline{c_1} = x$) and $\mathcal{F}(\mathcal{S}_1^{\mathcal{A}} \cup \mathcal{S}_1^{\mathcal{AN}} \vdash c_1) \geq f(x)$.

The first condition spells out that an analogical argument may attack a standard argument. This certain characteristic corresponds to the investigation in (Waller 2001), where analogical arguments can be used for persuasion. For instance, saying "geese can quack because they are similar to ducks" may effect the belief's changing on the opponent if no evidences to falsify the argument can be shown up. To put it more precisely, an opponent can be persuaded to believe a conclusion and that conclusion is inherently subject to be challenged. Hence, the burden of proof is shifted back to an opponent after he/she is persuaded to believe in that conclusion.

The second condition associates with another circumstance *i.e.* an analogical argument can attack an assumption only if the argument has been labeled with the number higher than or equal to the number associated with the assumption. This way of treatment is not used in (Waller 2001; Walton, Reed, and Macagno 2008).

Example 4.7. Figure 2 illustrates an overall ABA$^{(\mathfrak{p})}$ framework for the running example. According to the figure, the framework uses sim^{π} and π_0 as concrete instances of $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}}$ and $\mathfrak{p}$, respectively. The figure also uses $\overset{\mathfrak{p}}{\nsim}_{\mathcal{T}}$ to indicate 'being not similar under preference context $\mathfrak{p}$ w.r.t. $\mathcal{T}$'. The following suggests two arguments which can be constructed from the framework.

- $\{goose(bird_2, sound_2)\} \cup \left\{duck \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} goose, ?(quack(sound_2), duck, goose)\right\}$ $\vdash quack(sound_2)\}$ representing "$sound_2$ created by $bird_2$ is quack sound because $bird_2$ is a goose and geese are similar to ducks";
- $\emptyset \vdash honk(sound_2)$ representing "$sound_2$ is honk sound".

Hence, the second argument attacks the first argument. It is also worth observing that, in this case, varying each choice of $\otimes$ does not effect on the attack relation between these two arguments even though the degree of an

$$
\begin{array}{ll}
\mathcal{R}: & quack(B) \leftarrow duck(A,B); \quad honk(B) \leftarrow cord(A,B), built\_for\_honk(A); \\
 & cord(cord_1, sound_2); \quad built\_for\_honk(cord_1) \\
\mathcal{A}: & duck(bird_1, sound_1); \quad goose(bird_2, sound_2) \\
\overline{}: & \overline{duck(bird_1, sound_1)} = \neg duck(bird_1, sound_1); \\
 & \overline{?(duck(bird_2, sound_2), duck, goose)} = honk(sound_2); \\
 & \overline{duck \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} goose} = duck \overset{\mathfrak{p}}{\nsim}_{\mathcal{T}} goose \\
\mathcal{T}: & \mathsf{Duck} \sqsubseteq \mathsf{WaterBird}; \quad \mathsf{Goose} \sqsubseteq \mathsf{WaterBird} \\
\mathcal{M}: & duck^{\mathcal{M}} = \mathsf{Duck}; \quad goose^{\mathcal{M}} = \mathsf{Goose} \\
\overset{\mathfrak{p}}{\sim}_{\mathcal{T}}: & \mathsf{sim}^{\pi} \\
\mathfrak{p}: & \pi_0
\end{array}
$$

Figure 2. ABA$^{(\mathrm{p})}$ framework for the running example.

argument is changed. For example, if $\otimes_{\mathrm{min}}$ is used, then the degree of the first argument is equal to 0.5. On the other hand, if $\otimes_{\mathrm{mlt}}$ is used, then the degree of the first argument is equal to 0.25.

The following theorizes an observation which can be derived from Definition 4.6.

Theorem 4.8. *An analogical argument cannot attack a standard argument which does not use assumptions to support a claim.*

Proof. *Let argument $\mathcal{G}_1$ be defined as $\mathcal{S}_1^{\mathcal{A}} \cup \mathcal{S}_1^{\mathcal{AN}} \vdash c_1$ and argument $\mathcal{G}_2$ be defined as $\emptyset \vdash c_2$. We need to show that $\mathcal{G}_1$ cannot attack $\mathcal{G}_2$.*

Since $\mathcal{G}_2$ contains no assumptions, we conclude that $\mathcal{G}_1$ cannot attack $\mathcal{G}_2$. □

Theorem 4.8 shows that when an agent supports a claim from the grounded truth, it is impossible for other agents to persuade him/her by analogies. This corresponds to how analogical arguments are treated in practical reasoning.

Acceptability in ABA$^{(\mathrm{p})}$ and Its Link to Argumentation Scheme

Acceptability of Arguments in ABA$^{(p)}$

ABA$^{(\mathrm{p})}$ extends from ABA by equipping with predicate similarity and its attack definition is also extended for handling the degree of each argument and the preference between different types of arguments. Hence, ABA$^{(\mathrm{p})}$ can be considered as an instance of Dung's abstract argumentation. This implies that it can be used to determine whether a given claim is 'accepted' by a rational agent. In a sense of analogical argumentation, the claim could be a potential belief to be justified from analogies.

In order to determine the 'acceptability' of a claim, the agent needs to find an argument for the claim that can be defended against attacks from other arguments. To defend an argument, other arguments must be found and may need to be defended in turn (Dung, Kowalski, and Toni 2009). We formally define these characteristics as follows:

- A set of arguments Arg_1 *attacks* a set of arguments Arg_2 if an argument in Arg_1 attacks an argument in Arg_2;
- A set of arguments *Arg defends* an argument *arg* if *Arg* attacks all arguments that attack $\{arg\}$.

As in Dung's abstract argumentation, the notion of 'acceptability' can be formalized in many ways. In this work, we focus on the following notions:

- A set of arguments is *admissible* iff it does not attack itself and it attacks every argument that attacks it;
- An admissible set of arguments is *complete* if it contains all arguments that it defends;
- The least (w.r.t. set inclusion) complete set of arguments is *grounded*.

We observe that the correspondence between 'acceptability' of arguments and 'acceptability' of assumptions in $\mathrm{ABA}^{(\mathrm{p})}$ can be argued in the same way as in (Dung, Mancarella, and Toni 2007) for the link between ABA and AA. Hence, we know:

- If a set of assumptions *S* is admissible/grounded, then the union of all arguments supported by any subset of *S* is admissible/grounded;
- If a set of arguments *S* is admissible/grounded, then the union of all sets of assumptions supporting the arguments in *S* is admissible/grounded.

The above notion of acceptable sets of arguments provides a non-constructive specification. Now, we show how to turn the specification into a constructive proof procedure. The method we focus here is defined for a 'grounded' set of arguments and is extended from (Dung, Mancarella, and Toni 2007) for handling analogical arguments.

Informally, this constructive proof procedure is known as a *dispute derivation* which is defined as a sequence of transition steps from one state of a dispute to another. For each state, we maintain these following information. Component $\mathcal{P}$ maintains a set of (both standard and analogical) assumptions, which are used to support potential arguments of the proponent. Component $\mathcal{O}$ maintains multiple sets of assumptions, which are used to support all attacking arguments of the opponent. Component *D* holds a set of assumptions, which have already been used by the proponent. Component *C* holds a set of assumptions, which have already been used by the opponent and have been attacked by the proponent. Component *SP* maintains a set of triples holding an opponent's attacked assumption, a set of proponent's assumptions supporting a contrary of the attacked assumption, and a set of opponent's assumptions supporting the argument. Component SO maintains a set of triples holding a proponent's attacked assumption, a set of proponent's assumptions supporting the argument, and a set of opponent's assumptions supporting a contrary of the attacked assumption. In the following, we formally define the dispute derivation for a 'grounded' set of arguments.

Definition 5.1. Let an ABA$^{(\mathfrak{p})}$ is a 10-tuple $\langle \mathcal{L}_D, \mathcal{R}, \mathcal{A}, \overline{}, \mathcal{L}_T, \mathcal{T}, ^{\mathcal{M}}, \overset{\mathfrak{p}}{\sim}_{\mathcal{T}}, \mathfrak{p}, \mathcal{F} \rangle$. Given a 'patient' selection function[6], a 'grounded belief' dispute derivation of a defence set Δ for a sentence δ is a finite sequence:

$$\langle \mathcal{P}_0, \mathcal{O}_0, D_0, C_0, SP_0, SO_0 \rangle, \ldots,$$

$$\langle \mathcal{P}_i, \mathcal{O}_i, D_i, C_i, SP_i, SO_i \rangle, \ldots,$$

$$\langle \mathcal{P}_n, \mathcal{O}_n, D_n, C_n, SP_n, SO_n \rangle$$

where $\mathcal{P}_0 := \{\{\delta\}\}$, $D_0 := \mathcal{A} \cap \{\delta\}$, $\mathcal{O}_0 := \emptyset$, $C_0 := \emptyset$, $\mathcal{P}_n := \{\emptyset\}$, $\mathcal{O}_n := \emptyset$, $SP_0 := \emptyset$, $SO_0 := \emptyset$, $\Delta := D_n$, and for every $0 \leq i < n$, only one S in $\mathcal{P}_i$ or one S in $\mathcal{O}_i$ is selected, and:

(1) if S is selected in $\mathcal{P}_i$ and σ is selected in S, then
 (a) if σ is an assumption, then

$$\mathcal{P}_{i+1} := (\mathcal{P}_i \backslash \{S\}) \cup \{S \backslash \{\sigma\}\}, \quad \mathcal{O}_{i+1} := \mathcal{O}_i \cup \{\{\overline{\sigma}\}\},$$
$$\text{and} \quad SO_{i+1} := SO_i \cup \{\langle \sigma, S, \{\overline{\sigma}\} \rangle\}$$

 (b) else if there exists an inference rule $\sigma \leftarrow R \in \mathcal{R}$ such that $C_i \cap R = \emptyset$, then

$$\mathcal{P}_{i+1} := (\mathcal{P}_i \backslash \{S\}) \cup \{S \backslash \{\sigma\} \cup R\}, \quad D_{i+1} := D_i \cup (\mathcal{A} \cap R),$$
$$\text{and} \quad SP_{i+1} := (SP_i \backslash \{\langle \varphi, PA, OA \rangle\}) \cup \{\langle \varphi, PA \backslash \{\sigma\} \cup R, OA \rangle\}$$
$$\text{for any } \langle \varphi, PA, OA \rangle \in SP_i \text{ such that } \sigma \in PA$$

 and if $R \subseteq \mathcal{A}$, then further validation needs to be checked:
 for any $\langle \varphi, PA, OA \rangle \in SP_{i+1}$ such that $PA \cup OA \subseteq \mathcal{A} \cup \mathcal{AN}$, we have either $PA \subseteq \mathcal{AN}$ and $OA \subseteq \mathcal{A}$
 or $\mathcal{F}(PA) \geq \mathcal{F}(\varphi)$

 (c) else if $\sigma := P(t_1, \ldots, t_p)$ and there exists $\phi := Q(t_1, \ldots, t_p)$ such that $P^{\mathcal{M}} \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q^{\mathcal{M}} \in (0, 1]$, then

$$\mathcal{P}_{i+1} := (\mathcal{P}_i \backslash \{S\}) \cup \left\{ S \backslash \{\sigma\} \cup \left\{ P \overset{\mathfrak{p}}{\sim}_{\tau} Q, ?(\sigma, P, Q), \phi \right\} \right\},$$

$$D_{i+1} := D_i \cup \left\{ P \overset{\mathfrak{p}}{\sim}_{\tau} Q, ?(\sigma, P, Q) \right\} \cup (\mathcal{A} \cap \{\phi\}),$$

$$\text{and} \quad SP_{i+1} := (SP_i \backslash \{\langle \varphi, PA, OA \rangle\}) \cup \{\langle \varphi, PA \backslash \{\sigma\} \cup \{P \overset{\mathfrak{p}}{\sim}_{\tau} Q, ?(\sigma, P, Q), \phi\}, OA \rangle\} \text{ for any } \langle \varphi, PA, OA \rangle \in SP_i \text{ such that } \sigma \in PA$$

and if $\phi \in \mathcal{A}$, then the same validation as in Case 1.b is required

(2) If S is selected in $\mathcal{O}_i$ and σ is selected in S, then
 (a) if σ is an assumption, then
  (i) either σ is ignored, *i.e.*

$$\mathcal{O}_{i+1} := (\mathcal{O}_i \backslash \{S\}) \cup \{S \backslash \{\sigma\}\}$$

  (ii) or $\sigma \notin D_i$ and

$$\mathcal{O}_{i+1} := \mathcal{O}_i \backslash \{S\}, \quad \mathcal{P}_{i+1} := \mathcal{P}_i \cup \{\{\overline{\sigma}\}\}, \quad D_{i+1} := D_i \cup (\{\overline{\sigma}\} \cap \mathcal{A}),$$
$$C_{i+1} := C_i \cup \{\sigma\}, \quad \text{and } SP_{i+1} := SP_i \cup \{\langle \sigma, \{\overline{\sigma}\}, S \rangle\}$$

 (b) else if $A := \{R \mid \sigma \leftarrow R \in \mathcal{R}\}$ and $A \neq \emptyset$, then

$$\mathcal{O}_{i+1} := (\mathcal{O}_i \backslash \{S\}) \cup \bigcup_{R \in A} \{S \backslash \{\sigma\} \cup R\}$$

$$\text{and} \quad SO_{i+1} := (SO_i \backslash \{\langle \varphi, PA, OA \rangle\}) \cup \bigcup_{R \in A} \{\langle \varphi, PA, OA \backslash \{\sigma\} \cup R \rangle\}$$

$$\text{for any } \langle \varphi, PA, OA \rangle \in SO_i \text{ such that } \sigma \in OA$$

and further validation must be satisfied:
for any $\langle \varphi, PA, OA \rangle \in SO_{i+1}$ such that $PA \cup OA \subseteq \mathcal{A} \cup \mathcal{AN}$, we have
either $OA \subseteq \mathcal{AN}$ and $PA \subseteq \mathcal{A}$
or $\mathcal{F}(OA) \geq \mathcal{F}(\varphi)$

 (c) else if $\sigma := P(t_1, \ldots, t_p)$, $A := \{Q(t_1, \ldots, t_p) \mid P^{\mathcal{M}} \overset{\mathfrak{p}}{\sim}_{\tau} Q^{\mathcal{M}} \in (0, 1]\}$, and $A \neq \emptyset$, then

$$\mathcal{O}_{i+1} := (\mathcal{O}_i \backslash \{S\}) \cup \bigcup_{Q(t_1, \ldots, t_p) \in A} \{S \backslash \{\sigma\} \cup \{P \overset{\mathfrak{p}}{\sim}_{\tau} Q, ?(\sigma, P, Q), Q(t_1, \ldots, t_p)\}\}, \quad \text{and} \quad SO_{i+1} := (SO_i \backslash \{\langle \varphi, PA, OA \rangle\}) \cup$$

$$\bigcup_{Q(t_1, \ldots, t_p) \in A} \left\{ \left\langle \varphi, PA, OA \backslash \{\sigma\} \cup \left\{ P \overset{\mathfrak{p}}{\sim}_{\tau} Q, ?(\sigma, P, Q), Q(t_1, \ldots, t_p) \right\} \right\rangle \right\}$$

$$\text{for any } \langle \varphi, PA, OA \rangle \in SO_i \text{ such that } \sigma \in OA$$

plus, the same validation as in Case 2.b is required

(d) else $\mathcal{O}_{i+1} := \mathcal{O}_i \backslash \{S\}$ and

$$SO_{i+1} := SO_i \backslash \{\langle \varphi, PA, PO \rangle \mid \langle \varphi, PA, PO \rangle \in SO_i \text{ and } PO := S\}.$$

A dispute derivation can be seen as a way of representing a 'potential' winning strategy for a proponent to win a dispute against an opponent. The proponent starts by putting forward a claim whose acceptability is under dispute. After that, there are many possibilities as follows. The opponent can try to attack the proponent's claim by arguing for its contrary (*cf.* Case 1.a). The proponent argues for a non-assumption by using an inference rule (*cf.* Case 1.b). If an inference rule does not exist, the proponent can use an analogy to support the initial claim (*cf.* Case 1.c). Moreover, the proponent can select an assumption in one of the opponent's attacks and either ignores it because it is not selected as a culprit (*cf.* Case 2.a.i) or decides to counter-attack it by showing its contrary (*cf.* Case 2.a.ii). Otherwise, the opponent can argue for a non-assumption by using either an inference rule (*cf.* Case 2.b) or an analogy (*cf.* Case 2.c). Unfortunately, the opponent may not have even a reason to argue for it (*cf.* Case 2.d). In addition, every attacking argument of the opponent to the proponent's claim is maintained inside *SO, i.e.*, $\langle \sigma, S, \{\overline{\sigma}\} \rangle$ is read as "assumption σ in a set of proponent's assumptions S is attacked by a set of assumptions $\{\overline{\sigma}\}$". Every attacking argument of the proponent to the opponent's claim is also maintained inside *SP, i.e.*, $\langle \sigma, \{\overline{\sigma}\}, S \rangle$ is read as "assumption σ in a set of opponent's assumptions S is attacked by a set of assumptions $\{\overline{\sigma}\}$".

We give an informal dispute derivation for the running example.

Example 5.2. Consider an ABA$^{(\mathfrak{p})}$ given in Figure 2 and let $\otimes_{\min}$ be used. Table 2 shows that there does not exist a grounded belief dispute derivation for $quack(sound_2)$, where[7] $\heartsuit$, $\clubsuit_1$, $\clubsuit_2$, $\clubsuit_3$, $\clubsuit_4$, $\spadesuit_1$, and $\spadesuit_2$ denote $\left\{d \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} g, ?(d(b_2, s_2), d, g), g(b_2, s_2)\right\}$,
$\langle ?(d(b_2, s_2), d, g), \heartsuit, \{h(s_2)\} \rangle$, $\langle ?(d(b_2, s_2), d, g), \heartsuit, \{c(c_1, s_2), bfh(c_1)\} \rangle$,
$\langle ?(d(b_2, s_2), d, g), \heartsuit, \{bfh(c_1)\} \rangle$, $\langle ?(d(b_2, s_2), d, g), \heartsuit, \emptyset \rangle$,
$\langle g(b_2, s_2), \left\{d \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} g, g(b_2, s_2)\right\}, \{\neg g(b_2, s_2)\} \rangle$, and
$\langle g(b_2, s_2), \left\{d \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} g, g(b_2, s_2)\right\}, \left\{d \overset{\mathfrak{p}}{\nsim}_{\mathcal{T}} g\right\} \rangle$, respectively.

At step 2, the proponent ($\mathcal{P}$) has completed the construction of an argument for $q(s_2)$ supported by $\heartsuit$, saying that "s_2 is a quack sound because goose b_2 makes s_2 and geese are similar to ducks". At step 3, the opponent ($\mathcal{O}$) has decided to attack on assumption $?(d(b_2, s_2), d, g)$ by showing its contrary $h(s_2)$. This argument is fully constructed at step 6, in which no assumptions have been used. Nonetheless, this attacking argument needs to be checked at SO_6 if it satisfies the requirements of argument from analogy.

Table 2. A grounded belief dispute derivation for $quack(sound_2)$.

Step	$\mathcal{P}$	$\mathcal{O}$	D	C	SP	SO
0	$\{\{q(s_2)\}\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
1	$\{\{d(b_2, s_2)\}\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
2	$\{\heartsuit\}$	$\emptyset$	$\heartsuit$	$\emptyset$	$\emptyset$	$\emptyset$
3	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g, g(b_2, s_2)\right\}\right\}$	$\{\{h(s_2)\}\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_1\}$
4	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g, g(b_2, s_2)\right\}\right\}$	$\{\{c(c_1, s_2), bfh(c_1)\}\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_2\}$
5	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g, g(b_2, s_2)\right\}\right\}$	$\{\{bfh(c_1)\}\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_3\}$
6	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g, g(b_2, s_2)\right\}\right\}$	$\{\emptyset\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_4\}$
7	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g\right\}\right\}$	$\{\emptyset, \{\neg g(b_2, s_2)\}\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_4, \spadesuit_1\}$
8	$\left\{\left\{d \overset{\mathfrak{p}}{\sim}_{\tau} g\right\}\right\}$	$\{\emptyset\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_4\}$
9	$\{\emptyset\}$	$\left\{\emptyset, \left\{d \overset{\mathfrak{p}}{\not\sim}_{\tau} g\right\}\right\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_4, \spadesuit_2\}$
10	$\{\emptyset\}$	$\{\emptyset\}$	$\heartsuit$	$\emptyset$	$\emptyset$	$\{\clubsuit_4\}$

Since it satisfies, step 6 is valid. Finally, no arguments of the proponent can defend the opponent's argument at step 10, this dispute derivation fails.

With an analogous manner, we can find a grounded belief dispute derivation of $\{d(b_1, s_1)\}$ for $q(s_1)$ with three transition steps.

Relationship to Argumentation Scheme for Argument from Analogy

Since $\text{ABA}^{(\mathfrak{p})}$ extends from ABA with the capability for supporting the conclusion from similarity premises, the notion of argument trees in $\text{ABA}^{(\mathfrak{p})}$ can be also used to display the structural relationships between conclusions and assumptions including standard assumptions and analogical assumptions. Figure 3 illustrates an example of argument trees for arguments discussed in Example 4.7. The figure uses a rounded rectangle for indicating an argument tree, a number floating near a rounded rectangle for indicating an annotated degree of that entire argument, a number floating near an assumption for indicating an annotated degree of that assumption, and a dashed arrow for indicating an attack. For example, the top rounded rectangle shows the structural relationship of argument "$sound_2$ created by $bird_2$ is quack sound of ducks because ducks are similar to geese and we know that $bird_2$, which is a goose, creates $sound_2$" whereas the bottom rounded rectangle shows the structural relationship of argument "$sound_2$ is honk sound because it is created from $cord_1$ and that cord is built for honk". The figure also depicts that the bottom one attacks the top one.

Ones may observe that the structural relationship represented by an argument tree directly corresponds to the relationship between premises and a conclusion used in the argumentation scheme. That is, a similarity

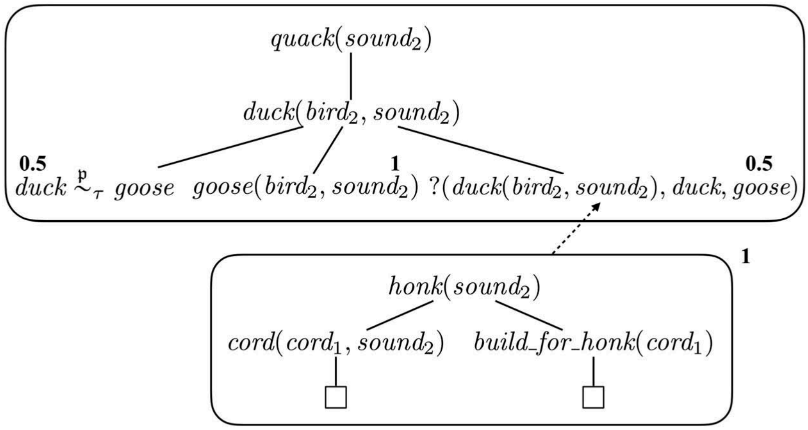

Figure 3. An example of argument trees and their relationship.

premise appears as an assumption of the form $P \overset{\mathfrak{p}}{\sim}_{\mathcal{T}} Q$ and a base premise appears as either an assumption in $\mathcal{A}$ or an inference rule with the empty body in $\mathcal{R}$. They appear as nodes in an argument tree. A conclusion drawn from the use of the argumentation scheme is represented as a parent of those nodes in an argument tree. This structure clearly explains the relationship indicated in the argumentation scheme.

The critical questions can also be captured in ABA$^{(\mathfrak{p})}$. Let us repeat that page 2 writes down each critical question (CQ) matching the scheme argument from analogy. Firstly, asking CQ1 is captured by the provability of a claim *i.e.* a backward deduction from a claim to its supporting assumptions. Secondly, CQ2 and CQ3 are formalized by the use of a similarity measure together with a supplied terminological formalism. Since similarity measure of concepts identifies the degree of commonalities, it automatically models the questions. Lastly, the notion of counter-analogies can be also modeled by the construction of arguments from another analogies drawing the contrary of the defeasible condition of the former argument.

Argumentation schemes employ the idea of asking critical questions to evaluate the acceptability of generated arguments. In ABA$^{(\mathfrak{p})}$, we evaluate by employing the notion of attack together with a semantics of argumentation framework (Dung 1995) insisting that sets of acceptable arguments do not attack themselves and counter-attack all the opponent's arguments (*aka.* admissible sets of arguments).

Comparison with Related Works

There were attempts on modeling analogical reasoning including our recent work (Racharak et al. 2016, 2017) in which their results are continued to study in this work. We note that both formalized the scheme argument from analogy and provided a logical language which enables finding analogical conclusions. On the other hand, (Racharak et al. 2016) extended syntax and argumentative features of DeLP for handling analogical arguments whereas (Racharak et al. 2017) translated the logical language to the represented answer set program and an answer set solver would be used to compute analogical conclusions. As (Racharak et al. 2016) extended DeLP, this work differs to (Racharak et al. 2016) in the structure of an argument's notion. Another difference is that (Racharak et al. 2016) is more computationally oriented and has restricted expressiveness whereas ABA$^{(\mathrm{p})}$, like ABA, is a more general framework for analogical argumentation. With (Racharak et al. 2017), it is worth observing that their definition of knowledge base can be captured by an ABA$^{(\mathrm{p})}$ framework. That is, a logic program $\mathcal{LP}$ is mapped to an ABA component, $\mathcal{O}$ is a concrete instance of $(\mathcal{L}_T, \mathcal{T})$, and $\overset{\pi}{\sim}_\tau$ is an abstract instance of $\overset{\mathfrak{p}}{\sim}_\tau$. However, the development in (Racharak et al. 2017) ignored analogical degrees in their computational method. We have completed that part and generalized the approach in this work.

A similar attempt to (Racharak et al. 2016, 2017), *i.e.*, combing rules and similarities, was proposed in (Sun 1995). In that work, a two-level connectionist model was developed. The first level (called CL) had one node for each domain concept whereas the second level (called CD) had fine-grained features in which all domain concepts could be decomposed to. Characteristics of similarity measures (denoted by $\sim$ in (Sun 1995)) was also discussed and the formula based on the above two-level model was proposed for concepts A, B as: $A \sim B = (|F_A \cap F_B|)/(|F_B|)$ where F_A, F_B are features defined in CD. It is worth observing that those two levels and similarity formula can be represented as $(\mathcal{L}_T, \mathcal{T})$ and $\overset{\mathfrak{p}}{\sim}_\tau$, respectively. However, how defeasible conditions and the notion of relevence should be handled was not discussed concretely.

In (Goebel 1989), the form of analogical reasoning was cast as hypothetical reasoning as: $\textit{source knowledge} \cup \textit{target knowledge} \cup \textit{equality assumptions} \models$ *conclusions* where equality assumptions can be viewed as similarity between the source and the target. If there were many equality assumptions, certain explicit preferences, *e.g.*, the highest number of shared properties, were used. However, the defeasible conditions and the notion of relevance were also not concretely discussed. It is also worth observing that source knowledge and target knowledge can also be recast in $(\mathcal{L}_T, \mathcal{T})$ and the criterion for forming equality assumptions can be made explicitly in $\overset{\mathfrak{p}}{\sim}_\tau$.

Case-based reasoning (CBR) can also be viewed as a form of analogical reasoning. In CBR, dimensions and factors are used for comparing cases and the decision in the precedent case is then taken as the decision into the current case. Examples of CBR systems are HYPO (Ashley 2006) and

CATO (Aleven 1997). With ABA$^{(p)}$, CBR can be recast by consisting the rules: $c_i \leftarrow f_1, \ldots, f_n$ in $\mathcal{T}$, the rules: $p_i \leftarrow c_i$ in $\mathcal{R}$, and similarity between two cases c_i are measured from their common features f_i.

Comparing this work with defeasible reasoning formalism, particularly Nute's d-Prolog (Gabbay, Hogger, and Robinson 1998, pp.353–396), different forms of rules were introduced *viz.* strict (unchallengeable) rules, defeasible (challengeable) rules, and defeater (exceptionable) rules. Examples of strict rules, defeasible rules, and defeater rules are "all penguins are bird", "birds normally fly", and "sick birds do not fly", respectively. Like ABA, inference rules in ABA$^{(p)}$ can be seen as strict rules and a simple transformation (as used in Theorist (Poole 1988)) can be employed to convert defeasible rules into strict rules with assumptions. Moreover, we may observe that ABA$^{(p)}$ does not need to supply with defeater rules since it can find counter-arguments, including counter-analogies, among arguments it is able to build.

Ones may would like to compare between ABA$^{(p)}$ and an abstract framework of argumentation equipped with a preorder relation *e.g.* preference-based argumentation framework (PAF) introduced in (Amgoud and Cayrol, 2002). Formally, a PAF is a triple $\langle Args, Attack, \preceq \rangle$ where $Args$ is a set of arguments, $Attack$ is an attack relation, and $\preceq$ is used to define a 'defeat' relation on each attack. It is not difficult to observe the correspondence between an ABA$^{(p)}$ framework and a PAF framework. Informally, each argument tree in ABA$^{(p)}$ is mapped to an argument in $Args$ and an attack in ABA$^{(p)}$ between argument trees is mapped to a defeat relation, in which the usage of an argument's degree and the preference on analogical arguments can be captured in a preorder relation. Their further theoretical relationship is left for future work.

Discussion and Future Work

This paper introduces a structured argumentation framework called ABA$^{(p)}$, which formalizes the argumentation scheme for argument from analogy. ABA$^{(p)}$ offers ways to encode the pattern of reasoning in argument from analogy and its critical questions, where concepts (or states of affairs) are represented by predicates in an underlying language and are defined by a particular terminological formalism. Its underlying mechanism consists in four mainstreams, *viz.* an ABA framework, a terminology, and a concept similarity under preferences, and a preference context. When no assumptions are available to construct an argument tree, additional assumptions can be constructed from the use of a similarity measure w.r.t. a terminology and a preference context. In other words, it draws a connection between two different formalisms, *i.e.*, inference rules and terminological sentences, for dealing with analogical argumentation.

$\text{ABA}^{(\text{p})}$ is meant to be a general framework for analogical reasoning. Thus, other notions apart from an ABA framework are also remained in general. For instance, ones may express a terminology as inference rules in $\mathcal{T}$ underlying a language $\mathcal{L}_T$ and $\overset{\mathfrak{p}}{\sim}_{\mathcal{T}}$ may be defined as a proportion of common features to different features as discussed in Section 6. In this work, we exemplify how ones can use a particular description logic to express terminological formulae and our recent developed measure sim^{π} is also demonstrated. One benefit of using description logics is that their expressivity and computational complexities were clearly studied (Baader et al. 2007).

Like ABA, all semantic notions for determining the acceptability of arguments in AA also apply to arguments in $\text{ABA}^{(\text{p})}$. Thus, we investigate a constructive proof procedure for determining a grounded set of assumptions in this work. Since different agents may value analogies for their reasoning unequally, we also study how each choice of operator $\otimes$ can influence different types of agents in analogical reasoning. Concerning other semantic notions of acceptability, this becomes an obvious future work to investigate on a dispute derivation for them and to further study how each semantic notion contributes to analogical argumentation in practice.

Other future directions are as follows. Firstly, we intend to apply the framework in some practical domains where analogical reasoning is extensively used, *e.g.*, in clinical practices. In the clinical domain, many terminologies do exist and are represented in description logics *e.g.* SNOMED CT and GO. The remaining tasks will be then encoding the actual methods of medical experts in terms of inference rules. Secondly, in light of argumentation schemes (Macagno, Walton, and Tindale 2017) developed some inferential structures and defeasibility conditions for analogical arguments; thus, we aim at investigating if such inferential structures can be captured by $\text{ABA}^{(\text{p})}$. Finally, we are interested to theoretically study the relationship between other instances of PAF and $\text{ABA}^{(\text{p})}$ in the viewpoint of analogical argumentation.

Notes

1. We use inference rule schemata, with variables starting with capital letters, to stand for the set of all instances obtained by instantiating the variables so that the resulting premises and conclusions are sentences of the underlying language. For simplicity, we omit the formal definition of the language underlying our examples.
2. In DL, a structure $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty domain and $\cdot^{\mathcal{I}}$ is an interpretation function mapping each concept name A to $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and each role name r to $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, is said to be a *model* of TBox $\mathcal{T}$ if it satisfies all formulae in the obvious way *i.e.* $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$ for all formulae $A \sqsubseteq C$, $A^{\mathcal{I}} = C^{\mathcal{I}}$ for all formulae $A \equiv C$, and $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$ for all formulae $r \sqsubseteq s$ in $\mathcal{T}$.

3. See Definition 4.3, for its formal definition.
4. If $p = 0$, both P and Q are called *propositions*.
5. The choice of $\overset{p}{\sim}_{\tau}$ also contributes to the type of a rational agent. That is, different concrete measures may have different skepticism. However, the definition only pays attention to how gullible is contributed from $\mathcal{F}$.
6. A patient selection function always prefers a non-assumption to an assumption in its selection.
7. Obvious abbreviations are used here for the sake of succinctness.

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