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Bi-level Decision-making Modeling for an Autonomous Driver Agent: Application in the Car-following Driving Behavior

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ABSTRACT

Road crashes are present as an epidemic in road traffic and continue to grow up, where, according to World Health Organization; they cause more than 1.24 million deaths each year and 20 to 50 million non-fatal injuries, so they should represent by 2020 the third leading global cause of illness and injury. In this context, we are interested in this paper to the car-following driving behavior problem, since it alone accounts for almost 70% of road accidents, which they are caused by the incorrect judgment of the driver to keep a safe distance. Thus, we propose in this paper a decisionmaking model based on bi-level modeling, whose objective is to ensure the integration between road safety and the reducing travel time. To ensure this objective, we used the fuzzy logic approach to model the anticipation concept in order to extract more unobservable data from the road environment. Furthermore, we used the fuzzy logic approach in order to model the driver behaviors, in particular, the normative behaviors. The experimental results indicate that the decision to increase in velocity based on our model is ensured in the context of respecting the road safety.

Introduction

Road crashes are present as an epidemic in road traffic and continue to grow, where, according to World Health Organization (WHO [2015](#page-22-0)), they cause more than 1.24 million deaths each year and 20 to 50 million non-fatal injuries, so they should represent by 2020 the third leading global cause of illness and injury. In this context, the Car-Following (CF) behavior that represents the basic unit to ensure the longitudinal movement of vehicles is an important issue in terms of road safety, where, according to Distner [\(2009](#page-21-0)), it alone represents nearly 70% of road accidents, which they are caused by the incorrect judgment of the driver to keep a sufficient safety distance.

According to various research projects, such as (Papageorgiou [1991](#page-22-1); Sameh, Alexis, and Stephane [2002](#page-22-2)), the safety problems are generated by

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individual driver errors. In addition, René et al. ([2008\)](#page-22-3) indicated that human factors appear in more than 90% of road accident. As a result, the driver behaviors become a major concern for researchers (Benjamin et al. [2017](#page-20-0); Victor, Christelle, and L.M. [2012](#page-22-4)). In this context, according to various research studies, such as (Golias, Yannis, and Antoniou [2001](#page-21-1)), driver behavior is classified into two types, normative behavior, and non-normative behavior. The first type ensures that the driver behavior adopted leads the driver to respect the Highway Code, thus increasing the road safety rate. On the other hand, the second type eliminates the road safety objective, by giving priority to reducing travel time with violent driving. In this context, the simulation is used as a perfect solution to study human behaviors, whose objective is to propose solutions that reduce the enormous rate of accidents caused by the human factor.

For the imitation of human behavior, we are interested in the decisionmaking of a driver agent based on CF driving behavior, which is the most accidental behavior. In this context, the decision-making during this driving behavior has two aspects: how far away the follower conductors allow a leading vehicle to get closer with an acceptable distance, and how they control the velocity according to the leading vehicle stimulus. In this context, the velocity has been identified as a key risk factor, influencing both on the road safety and on the travel time, where decision-making for the velocity value to be adopted in the near future is expressed using mathematical techniques, such as differential equations. Such solutions are sometimes sufficient but are not acceptable when the purpose of the simulation is to mimic the actual behavior of human drivers. In fact, the decision-making ensures the production of road phenomena that are the result of an individual decision error influenced by driver behavior, thereby directly influencing on road safety. However, mathematical techniques do not ensure the production of road phenomena, but they ensure reproduction, which does not ensure the imitation of human behaviors.

The modeling and implementation of the CF microscopic driving behavior require a technology that guarantees autonomy, responsiveness, adaptability, and interaction. The technology of software agents meets these criteria perfectly and is positioned as an appropriate solution to simulate driver behaviors. Thus, we used this technology to model the drivers during the simulation.

In short, the main contributions of this paper are:

- (1) Designing a decision-making model that uses the fuzzy one to know the velocity and the safety distance, which will be applied by the follower driver agent to react to the environment state in near future.
- (2) Reporting the simulation experimentation rests on actual instances was treated by the American Federal Highway Administration Program called

the Next Generation SIMulation computer (NGSIM [2005\)](#page-22-5), where the used data were collected from the stretch of US101 highway (Hollywood Highway) in Los Angeles, California; in addition to a comparison against the reference work of Gipps ([1981\)](#page-21-2), and a recent work of Yang et al. [\(2014\)](#page-22-6).

In the second section, we present the related works of decision-making with CF driving behavior. In the third section, we illustrate the details of our proposition of a decision-making model. In the fourth section, we present the experimental results. Finally, we finish this paper with conclusions and future works.

Related Work

The study of the CF behavior began in the 1950s (Chandler, Herman, and Montroll [1958](#page-21-3); Gazis, Herman, and Potts [1959](#page-21-4)) and this in the more general context of road traffic analysis. These first models open the way to the longitudinal control modeling. Indeed, the CF models can be classified under two categories. The first one is the classic CF models, where over time it updates the state variables by using equations. The second one is based on artificial intelligence.

Analytical Models

The classic CF models contain several categories, such as the Stimulus-response models. These precursor models are based on the assumption that the triggering of driver actions (acceleration, braking) is caused by external stimuli such as the variation of an inter-vehicle distance. The advantage of this model lies in its simplicity in terms of calculations, implementation, and estimation of its parameters in reduced numbers. However, it assumes that drivers have the ability to assess the relative velocity between the two vehicles. Gazis, Herman, and Potts ([1959\)](#page-21-4) supposed that the term of sensitivity is inversely proportional to the distance between the lead vehicle and its follower. Thus the sensitivity increases as the relative distance between the two vehicles decrease, giving a better reactivity of the driver and this at the slightest change in relative velocity. Conversely, the driver reacts little when he is away from the vehicle followed. Newell [\(2002](#page-22-7)) also proposed a simplified "Stimulus-Response" type model based solely on the estimation of distance indices. The driver behavior of the follower vehicle is summarized in the reproduction of that of the vehicle followed in reaction time and desired inter-vehicle distance.

The safe and optimal distance models are one of the classic CF models. According to this approach, the driver is supposed to control his velocity so as to maintain between his vehicle and the one preceding it a distance to avoid any risk of collision in case of sudden braking of the vehicle followed. Gipps [\(1981](#page-21-2)) 1160 $\left(\rightarrow\right)$ A. BENNAJEH ET AL.

proposed a model based on the fundamental principle of dynamics. In this model, the driver has two velocity controls which he chooses for safety the one with the lowest value. One ensuring compatibility with its desired velocity and acceleration (not taking into account other traffic vehicles), the other guaranteeing it to avoid any collision with the vehicle followed in case of untimely braking. Like collision avoidance type models, Bando et al. [\(1995\)](#page-20-1) proposed that the driver regulates his velocity not on the basis of that of the vehicle he follows, but on a velocity which guarantees him the case of sudden braking to avoid any risk of contact with the vehicle followed. In the same context of Bennajeh et al. ([2016a](#page-20-2), [2016b](#page-20-3))) proposed a decision-making model for the CF behavior problem by taking into account the road safety and the reduction of travel time. Other safe distance models have also been developed, such as (Broqua et al. [1991;](#page-21-5) Qiang et al. [2011;](#page-22-8) Yang et al. [2014\)](#page-22-6).

Models Based on Artificial Intelligence

Human behavior is a very complex paradigm, which requires more than mathematical equations to ensure a perfect imitation. In this case, the using of artificial intelligence to resolve this problem is presented as a perfect solution. Among the artificial intelligence models developed in the past two decades, the fuzzy logic (FL) approach is presented as the best approach to ensure the imitation of the logic of the human thinking, especially with adopting the CF driving behavior, since drivers decide and act based on their experience, logic, and judgments. In this context, to examine the qualitative and unclear decision of the drivers, several models of the FL approach have been developed and used in traffic studies. Kikuchi and Chakroborty [\(1992\)](#page-21-6) applied FL rules to model the CF behavior for the first time. After Kikuchi and Chakroborty ([1992\)](#page-21-6) model, many other CF models were developed based on FL rules, such as (Chakroborty and Kikuchi [1999](#page-21-7); Das et al. [1999;](#page-21-8) Gao, Hu, and Dong [2008;](#page-21-9) Gonzalez-Rojo et al. [2002;](#page-21-10) Hao, Ma, and Xu [2016;](#page-21-11) Hatipkarasulu and Wolshon [2003;](#page-21-12) McDonald, Wu, and Brackstone [1997](#page-22-9); Won et al. [2007](#page-22-10); Zheng and McDonald [2005](#page-22-11)).

Discussion of Existing Works

The above works examined the existing microscopic models of CF driving behavior in two broad categories: classic and artificial intelligence models. Their concepts and properties have been highlighted by providing some examples of their applications. Indeed, the artificial intelligence models based on complex algorithms, which make them more complicated to understand and use compared to the classic models that are based on reasonable equations. In fact, the classic models play the most important role in many existing traffic simulation models. However, these latter do not consider the examination of the driver behavior when making decisions, which is the most important point of the intelligence artificial models. In fact, the classic models consider the driver behavior by measuring the characteristics of the vehicle, such as spacing, velocity, and acceleration, it can be accepted for normal traffic conditions, but in severe situations such as congestion, driver behavior needs complex modeling. In this context, the models based on the artificial intelligence resolved this complex problem, but these existing models, always based on a single objective, where, it does not exist a model that provides the combination of more than one objective at the same time of decision making, such as, the increasing of the road security and the reducing of the travel time, which is the main objective of this article.

Main Idea and Motivations

The phenomena of road traffic, such as congestion, occupation of traffic lanes and road accidents. They are concluded from the interactions between the drivers and the individual practices of these later. However, mathematical models work according to the equations that guarantee the accuracy of the decision to be made. Thus, these models do not provide the production of traffic phenomena, but they ensure reproduction, which influences the efficiency of the simulation.

Based on the aforementioned anomalies, modeling of making-decision is proposed in this paper to ensure a perfect imitation of human behavior with the CF driving behavior. Indeed, this modeling is based on a hybrid between the mathematical models in order to exploit the criterion of precision of these models, and the artificial intelligence-based models in order to exploit the generation of road phenomena based on behavioral approaches. Thus, we try to create a driving agent characterized by intelligence and autonomy during decision making, to ensure the production of road phenomena as a result of the decisions taken by him. In this context, we have incorporated anticipatory behavior to the follower driver agent during the decision making and depending on the CF behavior, in a continuous space and in discrete time, and with a set of driving rules that reflects the normative behavior of drivers.

Model: Decision-making Based on Bi-level Modeling

Overview and Motivations

Driving a vehicle involves moving around in a changing environment. To move, drivers support a set of interactions described by the constraints of other drivers' behavior, road infrastructure, and regulation. A driver often aims to minimize his travel time. So, he tries to reach his maximum psychological velocity, also called the desired velocity. by considering his current state (velocity, position, etc.) and various constraints imposed by his environment (other vehicles, infrastructures, etc.). Thus, decision-making during CF behavior has two aspects: 1162 (\Leftrightarrow) A. BENNAJEH ET AL.

Figure 1. The bi-level modeling schema.

how far away the follower drivers allow a leading vehicle as an acceptable approach distance, and how also the follower drivers control the velocity depending on the movements of the leader vehicles. In this context, the velocity has been identified as a key risk factor, affecting both road safety and travel time.

In short, reducing travel time results is translated by increasing the velocity, which reduces the safety distance. However, according to the road security rules, if there is an increase in the velocity, then it is mandatory to have an increase in the safety distance. Thus, this road security rule influences on the operation of the CF driving behavior, where, to chose the velocity that will be applied by the follower driver, a problem is posed during the simultaneously optimize of the two contradictory objectives: the maximization of the velocity and the maximization of the safety distance. [Figure 1](#page-6-0) presents the bi-level modeling schema.

Upper Module

Objective Function

The problem consists of finding the values of decision variables satisfying a set of constraints and optimizing a vector function. In this context, we start with the objective function "maximization of the Safety Distance to be Adopted (SDA)", which is represented by Equation 1.

$$
\text{Max SD A}_{x}(T+1) = \text{ASD}_{x}(T) + (V_{y}(T+1) \times t) (V_{x}(T+1) \times t) (1)
$$

Where, $ASD_x(T)$ is the Adopted Safety Distance by the follower vehicle X during the simulation time step T and which shows the distance between the follower vehicle X and the leader vehicle Y, t is the simulation time step duration,

 $V_x(T + 1)$ and $V_y(T + 1)$ are the decision variables respectively representing the velocities of the follower vehicle X and its leader vehicle Y in the near future $T + 1$.

Definition Domains of the Decision Variables

The decision variable of the upper-level $V_x(T + 1)$ of the objective function "maximization of the safety distance" represents the velocity that will be applied by the follower vehicle X. The definition domain of this variable is based on the characteristics of the vehicle to be used in the simulation. Thus, the definition domain is expressed as follows: \forall V_x(T + 1) (V_x(T + 1) \in [0; V_{max}]), where V_{max} represents the maximum velocity that the vehicle X can reach.

In the modeling of Bennajeh et al. ([2016a](#page-20-2)), the definition domain of this variable is limited to a small variation of the velocity of the simulation time step T, which forced them to eliminate the objective function "maximization of the velocity" of the leader vehicle Y, thus eliminating the anticipation of the velocity that will be applied by this vehicle. As a result, the risk of a collision is increased, especially with the sudden change in driving behavior of the leader vehicle. In this new modeling, knowing the velocity of the leader vehicle Y in the near future becomes a problem to be solved as that of the follower vehicle X. Thus, the modeling of the new definition domain of the decision variable $V_v(T + 1)$ is expressed as follows: $\forall V_v(T + 1) (V_v(T + 1) . [0; V_{max}])$.

Constraints

The decision making in our modeling is based on the CF behavior and the driver behavior, in particular the normative behavior, where, according to Golias, Yannis, and Antoniou [\(2001](#page-21-1)) a driver with a normative behavior is characterized by the respect of the code of the road, and that translates with the CF driving behavior by respecting the maximum velocity of the traffic area and respecting the safety distance, in order to avoid longitudinal collision problems. Thus, for the modeling of the constraints to be respected by the follower driver agent, it is necessary that these constraints translate the driver normative behavior based on the continuous change in the environment state. The goal in this step is to find a set of values to assign to decision variables, and that satisfied all constraints.

For the constraints of the objective function of the upper level, "the maximization of the safety distance", the inequalities 2 and 3 eliminate the sudden deceleration of the leader driver that influences on the road safety, where, the velocities to be applied by the follower vehicle X and the head vehicle Y must be greater or equal to zero.

$$
V_x(T+1) > = 0 \tag{2}
$$

$$
V_{y}(T+1) > = 0 \tag{3}
$$

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Lower Module

Objective Function

The objective function "maximization of the velocity", $V_x(T + 1)$ and V_y $(T + 1)$, which reflect respectively the objective of the follower driver agent X and the leader driver agent Y. The modeling of this objective function is illustrated by Equations 4 and 5.

$$
\text{Max } V_x(T+1) = V_x(T) + (A_x \times t) + m \tag{4}
$$

$$
\text{Max } V_{y}(T+1) = V_{y}(T) + (A_{y} \times t) + m \tag{5}
$$

Where, A_x and A_y are the decision variables representing the action or also called the driving behavior (acceleration, deceleration, maintain velocity) that will be adopted by the follower vehicle X and the leader vehicle Y, t is the simulation time step duration and m is the margin of precision that influences on the accuracy of the velocity. In fact, the first objective function, "maximizing the safety distance to be adopted", contains two decision variables $V_x(T + 1)$ and $V_y(T + 1)$ representing the velocities of the vehicles X and Y in the near future $T + 1$. In addition, the same decision variables are designed as the objective function, "maximization of the velocity". Therefore, this is why the modeling of our problem is bi-level modeling.

Definition Domains of the Decision Variables

The decision variable A_x of the objective function "maximization of the velocity" of the follower vehicle X, presents the action that will be adopted and can be summed up in one of the following driving behavior: acceleration, deceleration and maintain velocity. To ensure a realistic simulation, the deceleration and acceleration values of the decision variable A_x must be realistic. Therefore, the definition domain of the decision variable A_x is a continuous domain. In this context, the selection of the deceleration value for the decision variable A_x must be between d_{\min} and d_{\max} . Similarly, the selection of the acceleration value for the decision variable A_x must be between a_{min} and a_{max} . Also, the selection of the maintain velocity value for A_x must be between m_{min} and m_{max} . In short, according to the modeling of Bennajeh et al. [\(2016a](#page-20-2)), the definition domain of the behavior "maintain velocity" contains only the value zero. However, it is impossible to have a deceleration or acceleration value equal to zero, thanks to several factors which ensure the change of driving behavior by a slight deceleration or acceleration. Therefore, if the deceleration value or acceleration value is below the recognition threshold of a human driver, then the action is known as maintaining velocity. Thus, the new definition domain of the decision variable A_x is expressed as follows: $\exists A_x (A_x \in [d_{\text{min}}; d_{\text{max}}] \vee A_x \in [a_{\text{min}}; a_{\text{max}}] \vee A_x \in [m_{\text{min}}; m_{\text{max}}]).$

Indeed, the decision variable A_v has the same definition domain as the decision variable A_x . The modeling of the definition domain of A_y translates as following: $\exists A_y (A_y \in [d_{\text{min}}; d_{\text{max}}] \vee A_y \in [a_{\text{min}}; a_{\text{max}}] \vee A_y \in [m_{\text{min}}; m_{\text{max}}]).$

Constraints

To define the constraints of the objective function of the lower level, "the maximization of the velocity", the increase in the velocity is linked to the driver behavior, such as a driver with normative behavior ensures that the velocity that will be applied must lead to respect the Highway Code, which increases the rate of road safety. On the other hand, a driver who is characterized by non-normative behavior eliminates the objective of safety road, thus giving priority to the reduction of driving time by a violent increase in velocity. In this context, the decision making in our modeling is related to the driver normative behavior. Thus, it is necessary to have modeled the constraints which translate these normative behaviors, modeling, which is not based on mathematical equations. For this reason, the FL approach relates to the human reasoning flexibility and ensures the modeling of data imperfections by using qualitative and quantitative descriptions in which observations of drivers agents are expressed. This approach gives the possibility of modeling the constraints to be respected by the values of the decision variables A_x and A_y according to the linguistic rules. In addition, since the decision variables A_x and A_y are slaved to thresholds; it is then quite possible to use a fuzzy regulator to ensure this selection task, respecting the environment state, as well as the driver behavior. At this point, we replaced the analytical modeling of Bennajeh et al. [\(2016a\)](#page-20-2) with an input/output system consisting of a set of linguistic rules, a "black box" whose outputs predicted the action to be adopted (accelerating or decelerating or maintaining velocity) in the future.

For the extraction of the values for the decision variables A_v (T + 1) and A_v $(T + 1)$, which satisfy the constraints of the objective functions of the lower level, "the maximization of the velocity", we have adopted the anticipatory model of Bennajeh et al. [\(2018b](#page-21-13)).

Lower Level Fuzzy Constraints

The FL approach is used as a solution to ensure the imitation of human reasoning, using an expressive language closer to natural language and translated by qualitative and quantitative descriptions. Therefore, in the road traffic context, in particular with the CF driving behavior, we can estimate the velocity that respects the environment state and the objectives of the driver, according to his behavior, which is in our case a normative behavior. In this context, we used the anticipation approach defined by Bennajeh et al. [\(2018a,](#page-21-14) [2018b](#page-21-13))), which is based on the FL approach.

Bi-level Resolution Algorithms

Main Algorithmic Schema

Our bi-level modeling is composed by two problems, which are: the decision making of the safety distance that will be applied in the near future, which is

Figure 2. The main algorithmic schema.

presented in the upper level of our modeling, and the decision making of the velocity that will be applied in the near future, which is presented in the lower level of our modeling. For the resolution, we used the Genetic Algorithm (GA). Furthermore, we used the Tabu Search Algorithm in the lower level as it is defined by Bennajeh et al. ([2018b](#page-21-13)). Thus, the resolution of our modeling is based on two nested algorithms: the genetic algorithm and the Tabu search algorithm. In this context, [Figure 2](#page-10-0) presents the main algorithmic schema.

GA Module

Based on the studies on the use of meta-heuristics for micro-simulation models (Abhay, Alexander, and Steven [2017;](#page-20-4) Hollander and Liu [2008\)](#page-21-15) suggest that the Genetic Algorithm (GA) is the most popular optimization method. For example, it was used to calibrate the VISSIM traffic model to match the simulation results with the observed distributions obtained in the field (Kim, Kim, and Rilett [2005\)](#page-21-16). Also, the calibration approach based on GA is widely used, such as (Li, Liu, and Zhang [2009](#page-22-12); Manjunatha, Vortisch, and Mathew [2013](#page-22-13); Mathew and Radhakrishnan [2010](#page-22-14); Omrani and Kattan [2013](#page-22-15); Yu et al. [2006\)](#page-22-16). Therefore, the genetic algorithm is selected in this work for the calibration of the velocity that will be adopted in the nearest future.

Initial Population

According to Nicolas and Jean-Marc [\(1999](#page-22-17)), and Michalewicz and Janikov ([1991](#page-22-18)), a genetic algorithm is initially based on an initial population. Thus, the choice of the initial population of individuals strongly conditions the speed of the algorithm. If the position of the optimum in the state space is totally unknown, it is natural to randomly generate individuals by making uniform draws in each of the domains associated with the components of the search space, ensuring that the Individual products respect the constraints. On the other hand, if a priori information on the problem is available, it naturally seems natural to generate the individuals in a particular subdomain in order to accelerate the convergence. In the hypothesis where the management of the constraints cannot be done directly, the constraints are generally included in the criterion to be optimized in the form of penalties. It is clear that it is better when it is possible to generate only population elements respecting the constraints.

Based on this definition, we adopted an initial population generation mechanism capable of producing a population of individuals from a particular subdomain and which respects the constraints of our modeling. In this framework, for the values of the $V_x(T + 1)$ decision variable, we applied our anticipation strategy on the A_x decision variable, where we selected the best solution based on the defuzzification method that defined by Bennajeh et al. [\(2018b\)](#page-21-13) and the n-1 neighboring solutions. The same principle applied to the selection of n values for the decision variable $V_y(T + 1)$.

Evaluation Operator

Once the initial population was created, we made out the most promising individuals, those who will participate in improving our population. We have therefore assigned a rank which is presented as a quality index for each individual. The rank of an individual is the index of the population in which he has been noticed as being not dominated by no other individual. Then, we get out those who are not dominated by any other individual, we give them rank 1 and we make them stand out. Then, for the individuals who are not out of the lot, we repeat the operation, looking for those who are not dominated by any of the remaining individuals and we give them rank 2, and so on, until exhaustion of the population. Individuals considered the best are those of the lowest rank.

Selection Operator

We have adopted the method of binary tournament selection. This technique uses proportional selection on pairs of individuals and then chooses from among these pairs the individual who has the best quality. The quality of the individuals is treated in our modeling according to the Evaluation Criterion (EC) of Equation 6, where the individual with the lowest EC contains the highest quality.

$$
EC = |SDAx(T + 1)CSDx(T + 1)|
$$
\n(6)

Where $CSD_x(T + 1)$ is the calculated safety distance and has the evaluation function of the selected feasible solution. The calculated safety distance CSD_x $(T + 1)$ is expressed by Equation 7.

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$$
CSDx(T) = (RT + DT + AT) \times Vx(T + 1) + \frac{w}{2 \times G \times \rho \times Af \times Cd}
$$

$$
\times \ln\left(1 + \frac{\frac{\rho \times Af \times Cd}{2} \times Vx(T + 1)2}{(\eta \times \mu \times W) + (fr \times W \times \cos \theta) + (W \times \sin \theta)}\right)
$$
(7)

Where, RT is the reaction time, AT is the action time, DT is the decision time, W is the vehicle weight, ρ is the air density, G is the gravity speed, Af is the area of projection, η is the braking efficiency, Cd is the factor of the air resistance, µ is the coefficient of the friction, $θ$ is the road slope and fr is the factor of the decay. $V_{\rm v}(T + 1)$ is the adopted velocity during the simulation step T + 1. Indeed, the safety distance equation is defined by Chen and Wang [\(2007\)](#page-21-17).

Crossover Operator

The purpose of crossbreeding is to enrich the diversity of the population by manipulating the structure of chromosomes. In our work, we applied the elitist replacement with a probability of crossing P_c , where the selection of parents is treated randomly from the selected individuals with the selection operator. Thus, after the generation of the children, an internal evaluation in the crossing operator is started to check if the new individuals (children) meet all the constraints, otherwise, they will be eliminated and their parents will be sent back.

Mutation Operator

The role of this operator is to modify randomly the value of an individual component, ensuring thus the diversity of the population, and giving an equal chance for all the values of the search space. In this context, we applied the mutation operator with a probability of mutation P_m , where the selection of a solution is treated randomly from a value search space contained only values that satisfy all the constraints. The search space of a solution is concluded from the fuzzy rules.

Experimentation

Data Sets

To study the optimization results in the CF driving behavior, we adopted the vehicle trajectories of NGSIM [\(2005](#page-22-5)). Traffic data were collected on a stretch of highway US101 (Hollywood Highway) in Los Angeles, California. [Figure 3](#page-13-0) illustrates the US101 study area by using digital video cameras. The schematic drawing at the bottom of [Figure 3](#page-13-0) presents the ways of the highways and the location of the ramps on Ventura Boulevard and on Cahuenga Boulevard in the study area.

Figure 3. Simulation Zone (US101 Hollywood Highway.

The study reflects a total of 45 minutes of transcribed data from the digital video cameras were mounted, with which the Federal Highway Administration Program was processed a total of 11779 vehicle trajectories, which are recorded in a complete dataset.

Comparative Models

In addition to using the traffic data were collected from the Federal Highway Administration Program (NGSIM [2005\)](#page-22-5), and in the context to better discuss the performance of our model and for comparison, we selected two famous analytical models from the same family of car-following behavior as our proposed model, "Security distance models", which are: model of Gipps ([1981](#page-21-2)) and model of Yang et al. [\(2014](#page-22-6)).

Performance Metrics and Parameters Setting

In the study context of the simulation results, we used the evaluation indices, which are widely used in evaluating models of subroutines (for example Hamdar, Treiber, and Mahmassani [2009;](#page-21-18) Punzo and Simonelli [2005](#page-22-19)). The

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evaluation indices used are: Mean Error (ME), Mean Absolute Error (MAE), and Mean Square Error (MSE). The modeling of each is expressed respectively by Equations 8, 9 and 10.

$$
ME = \frac{1}{N} \sum_{n=1}^{N} (y_n^{real} - y_n^{sim})
$$
 (8)

$$
MAE = \frac{1}{N} \sum_{n=1}^{N} |y_n^{real} - y_n^{sim}|
$$
\n(9)

$$
MSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} |y_n^{real} - y_n^{sim}|}
$$
 (10)

Where, y_n^{real} is the nth data sample of the real vehicle, and y_n^{sim} is the nth data sample of the simulated vehicle. y_n^{real} and y_n^{sim} containing the adopted values for the decision variables A_x and $V_x(T + 1)$.

[Table 1](#page-14-0) presents the values of different simulation parameters used by the simulated vehicle 1720, where most of these values are collected from the model of Chen and Wang ([2007](#page-21-17)). However, the following simulation parameter values: d_{max} , d_{min} , a_{max} , a_{min} and V_{max} are based on the traffic data of NGSIM [\(2005\)](#page-22-5).

The results of simulation that are presented in the next section through the different tables and plots are calculated by the medium of 30 simulations performed. Furthermore, the number of individual adopted in the genetic algorithm is 100. Moreover, the number of iterations adopted by the GA is 46 and in the TSA is 50 iterations. In fact, the number of iterations adopted is not high, because according to Chen and Wang ([2007\)](#page-21-17), the decision time for an ordinary behavior driver is between 0.15 and 0.25 s, and according several simulations, these adopted numbers of iteration respect this constraint.

Comparative Results

In this context, to evaluate the effectiveness of simulation results, we adopted three criteria, which are:

- Imitation of human behavior.
- Reducing travel time.
- The guarantee of road safety.

Upper-level Results: Safety Distance

The evaluation criterion "imitation of human behavior" is reflected in the correspondence between the simulated sample and the actual samples. At this point, [Figure 4](#page-15-0) shows the trajectories generated as a function of space and time of 8 vehicles.

The curve with ID 1720 models the sample of the simulated vehicle based on a driver agent adopting our decision-making model, while the others represent actual vehicle samples. Therefore, the results of experiments conducted using the dataset of NGSIM [\(2005](#page-22-5)) to validate our optimization, indicate that the travel trajectory of the simulated vehicle 1720 is totally homogeneous with the actual travel trajectories in terms of deviation.

[Figure 5](#page-16-0) shows a comparison between the adopted safety distance (ASD) and the calculated safety distance (CSD). Indeed, the difference between the adopted safety distance and the calculated safety distance, presents the evaluation criterion (EC) to evaluate the homogeneity between the maximization of the travel velocity and the maximization of the safety distance that will be applied, thus allowing the

Figure 4. The spatiotemporal trajectories during travel of vehicles.

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Figure 5. Comparison of the safety distance adopted and the safety distance calculated between the 8 vehicles.

selection of the best solution that behaves as well as possible opposite the objectives of the driver agent and the continuous changes in the environment state.

According to the results in [Figure 5,](#page-16-0) we notice that with the results of the simulated driver 1720, there is a correspondence between ASD and CSD, which present the respect of the road security objective by respecting the CSD, and at the same time, ensure the reducing of the travel time, by increasing of the velocity. However, in the other graphs of the actual drivers, we notice that the ASD is bigger than CSD, which present exaggerate caution by eliminating the objective of reducing traffic time. Consequently, these results presents the success of our proposed model to ensure the correspondence between two objectives at a time during decision-making, which are: road safety by increasing the safety distance, in fact, a logical increase not an exaggerated increase, and the reduction of the travel time by the increase of the velocity, but by a logical increase also, which does not eliminate of the first objective.

Lower Level Results: Velocity

To study the simulation results against the aforementioned evaluation criteria, we have adopted an evaluation process, which consists in testing the error of the decision variable values $(A_x \text{ and } V_x(T + 1))$ used by the driver agent of the simulated vehicle 1720, in relation to the actual values used by a human driver of the vehicle 1717. In short, the vehicle 1717 does not appear in [Figures 4](#page-15-0) and [7](#page-18-0) because it has been replaced by the simulated vehicle 1720, to know the rate of correspondence between a vehicle driven by a driver agent based on our model and a vehicle driven by a human driver. The evaluation indices used are: Mean Error (ME), Mean Absolute Error (MAE), and Mean Square Error (MSE).

According to the evaluation indices values of the different models in [Table 2](#page-17-0), the mean error index (ME) on the decision variables (A_x and $V_x(T + 1)$) shows that the models of Gipps [\(1981\)](#page-21-2) and Yang et al. ([2014](#page-22-6)) tend to produce a lower velocity than our model. This illustration reflects the objective adopted by the driver agent based on our model, where, by increasing the velocity, the driver agent tries to reduce the travel time.

The Mean Absolute Error index (MAE) and the Mean Squared Error index (MSE) are two of the most common indices used to measure the accuracy of continuous variables. At this point, according to these two indices, the errors of our model are inferior to the models of Gipps ([1981](#page-21-2)) and Yang et al. ([2014\)](#page-22-6).

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	МE		MAE		MSE			
Models	v,	$H_{\rm Y}$	v.	A_{x}	v.	A_x		
Gipps (1981)	-0.29	-0.27	2.05	2.5	2.61	3.23		
Yang et al. (2014)	-0.17	-0.13	1.78	2.34	2.26	2.89		
Our model	-0.37	-0.01	1.16	1.04	1.56	1.42		

Table 2. Comparison of simulation results.

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Figure 6. Comparison of accelerations and velocities between a sample of the actual vehicle 1717 and a sample of the simulated vehicle 1720.

Figure 7. Comparison of the evaluation criterion (EC) between the 8 vehicles.

Thus, our modeling provides a better imitation of human behavior than the models of Gipps [\(1981\)](#page-21-2) and Yang et al. ([2014](#page-22-6)).

Based on the results of simulation, which illustrate through [Figure 6](#page-18-1), and which present the accelerations and the velocities adopted by vehicle 1720.

The graph (A) shows the action values adopted by the real vehicle 1717 and the simulated vehicle 1720, and the graph (B) presents the velocity values adopted by the real vehicle 1717 and the simulated vehicle 1720 during each simulation time step. Thus, as well as the results of our evaluation process in according in the two graphs (A) and (B), there is a high correspondence between the results of the real sample and the simulated sample.

Correspondence between the Upper and Lower Level Results

At this point, for the actual samples of vehicles 1706, 1699, 1690, 1685, 1673, 1667 and 1653, there is a significant gap between the two safety distances, where it obviously appears that it has no effect on road safety since CSD is wider than ASD. On the other hand, with the sample of the simulated vehicle 1720, there is a harmony between the calculated safety distance and the adopted safety distance, which reflects the increase in the mean velocity (MV) of the simulated vehicle 1720 compared to the actual vehicles in [Table 3,](#page-19-0) thus ensuring the reduction of travel time and taking into account the road safety.

[Figure 7](#page-18-0) contains 8 curves representing the evaluation criteria values of 8 vehicles during each simulation time step. The curve of the simulated vehicle 1720 contains the lowest EC, as shown in [Table 3,](#page-19-0) the Mean Evaluation Criterion (MEC) of the simulated vehicle is 1.097(m). However, MEC of the real vehicles varies between 9,606 (m) and 16,446 (m). Therefore, human drivers seek to ensure road safety, but in an inefficient way, since the distance between the safety distance adopted and the calculated safety distance has no effect on road safety, but it increases travel time. Thus, we conclude that the possibility of generating undesirable road phenomena (for example road congestion) from actual vehicle results is possible.

According to the simulation results at the final operator level "the selection of a final solution", if the adopted safety distance is very large compared to the calculated safety distance, then the priority is directed to the highest velocity, thus ensuring both, reducing travel time by reducing the gap between the two safety distances, as well as the guarantee of the road safety.

In short, the simulation based on our model shows that the criterion of road safety is always present in all states of the environment. On the other hand, the criterion of reducing travel time is related to the environment state. At this point, it obviously seems that the decision-making by a driver agent corresponds to the decision-making by a human driver with normative behavior.

Vehicles	MV(m/s)	MEC (m)
1720	10.124	1.097
1706	9.523	9.974
1699	9.21	9.606
1690	9.408	10.719
1685	9.356	13.861
1673	9.251	13.076
1667	9.185	16.446
1653	9.054	8.791

Table 3. The mean values of the evaluation criteria and the velocity of 8 vehicles.

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Conclusion

In this paper, modeling is proposed to ensure a perfect imitation of human behavior. In this context, our modeling of decision-making is based on the combination between the bi-level modeling and the FL approach, where, the upper level is solved by a set of constraints, and the lower level is solved by the FL approach defined by Bennajeh et al. [\(2018b\)](#page-21-13). For the resolution method, we used a genetic algorithm, where, according to the simulation results; we had solutions with high quality. However, in our decision-making modeling, there are two threats. Firstly, since our first objective is to ensure the imitation of human behaviors, in particular, the normative behaviors. Thus, there is a threat to drivers having non-normative behaviors. In this case, we plan to integrate the non-normative behaviors in order allow the simulated drivers to better react with the different behaviors of the neighbor drivers. Secondly, there is a threat in the FL approach, in particular, in the defuzzification, where, in some cases, more than a fuzzy set can appear to select value of the deceleration or acceleration or maintain velocity, which increases the search space. Thus, we plan to integrate a fourth input strategic variable presents the action of the leading vehicle.

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