



Oblateness Effects on Solar Sail in the Restricted Three-body Problem

Fatma M. Elmalky^{1*}, M. N. Ismail² and Ghada F. Mohamedien³

¹Department of Math, Faculty of Science (Girls), Al-Azhar Univesity, Egypt.

²Department of Astonomy and Meteorology, Faculty of Science, Al Azhar University, Egypt.

³National Research Institute of Astronomy and Geophysics (NRIAG), Helwan, Cairo, Egypt.

Authors' contributions

The three authors shared together through this work. Author MNI put the idea and the steps of the manuscript while authors FME and GFM did the computations, follow up the results and write the manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2020/v35i630289

Editor(s):

(1) Dr. Raducanu Razvan, Alexandru Ioan Cuza University, Romania.

Reviewers:

(1) Xu Deshui, Harbin Engineering University, China.

(2) Anindita Chakraborty, Bhilai Institute of Technology (BIT), India.

(3) Biqiao, University of Toronto, Canada.

Complete Peer review History: <http://www.sdiarticle4.com/review-history/59375>

Received: 26 May 2020

Accepted: 01 August 2020

Published: 13 August 2020

Original Research Article

Abstract

In the present work, the equations of motion of the solar sail are derived in the restricted three-body system. The dimensionless coordinates are used to obtain the solution of the problem. The Laplace transformations are used to solve these systems of equations to obtain the components of the solar sail acceleration. The motion about L2, L4 and its stability are studied under obalteness effects. The results obtained are in good agreement with previous results in this field. It is remarked that this model has special importance in space-dynamics to enabling spacecraft to do some maneuvers depends on the solar sail acceleration.

Keywords: Dynamical systems; lagrangian points; solar sail acceleration; laplace transforms; stability of equilibrium points; restricted three-body problem.

1 Introduction

In space dynamics, there are several systems like two-body, three-body, four- body, and N- body problem sunder considerations. It is very important for the dynamics of binary and multiple stars as well as the

*Corresponding author: E-mail: fatmasalem718@yahoo.com;

planetary systems. The restricted three-body problem (RTBP) has been considered as a basic dynamic model ever since the scientists have studied the orbital motion of solar sail [1,2]. Solar sail uses sunlight to generate propulsions in space by reflecting solar photons flux from a large mirror-like sail made of light weight: highly reflective polyimide film material. Solar sailing technology has widely investigated over the past decade [3,4]. It appears as a promising form of advanced spacecraft population, which can enable exciting new space-science mission concepts such as solar system exploration and deep space observation.

Reena Kumari [5] studied the equilibrium points in the restricted four-body problem with solar wind Drag. Solar sailing, an experimental method of spacecraft propulsion, depends on the radiation pressure from the Sun as a motive force. The idea of interplanetary travel by light was mentioned by Jules Verne [6].

There are many methods to treat the problem analytically and numerically one of these methods is the Laplace Transformations, it is a tool for solving linear differential equations by reducing the problem to an algebraic one [7,8]. Also, it takes care of initial conditions without the necessity of first determining the general solution and then obtaining the required solution from it. In this work, effect of the oblateness on the acceleration of solar sail is studied. A comparison of this case is done with the case obtained without oblateness.

Definition1: The Laplace transform of a function, $f(t)$, for $t > 0$ is defined by $F(s) = \int_a^b e^{-st} f(t) dt$

Where

- The improper integral must convergence (i.e. the limit exists and is finite) f or at least one value of s .
- t is real and called the time variable.
- s is complex and called frequency variable.

The resulting expression is a function of s , which symbol led by $F(s)$.

Definition2: The Inverse Laplace transform of the function $F(s)$ is given by $\frac{1}{2i\pi} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{st} F(s) ds$

Where the integral is taken over a line in the region of convergence and σ is large enough that $F(s)$ is defined for real $s \geq \sigma$.

This formula (integration in complex plane) is very difficult to apply directly. So we will use different approach.

2 Model of Solar Sail

The motion of the three body is studied, where its mass may vary insignificant compared with the masses of the other three-bodies and $m_1 > m_2 \gg m$. Fig. 1 illustrates the geometric of the problem, r_1 and r_2 are the position vectors from m_1 and m_2 to m respectively. The origin is considered at the center of mass of m_1 and m_2 which are called the primaries. The system Earth-Moon solar sail is used, where the masses of the Earth and Moon in the canonical system are given as $\mu_1 = \frac{M_1}{M_2+M_1} = 0.987871$ and $\mu_2 = \frac{M_2}{M_2+M_1} = 0.012151$ respectively.

The distance between the two primaries is unity.

$$r_1 = \sqrt{(x + \mu)^2 + (y - y_1)^2 + z^2}, \quad r_2 = \sqrt{(x + \mu - 1)^2 + (y - y_2)^2 + z^2} .$$

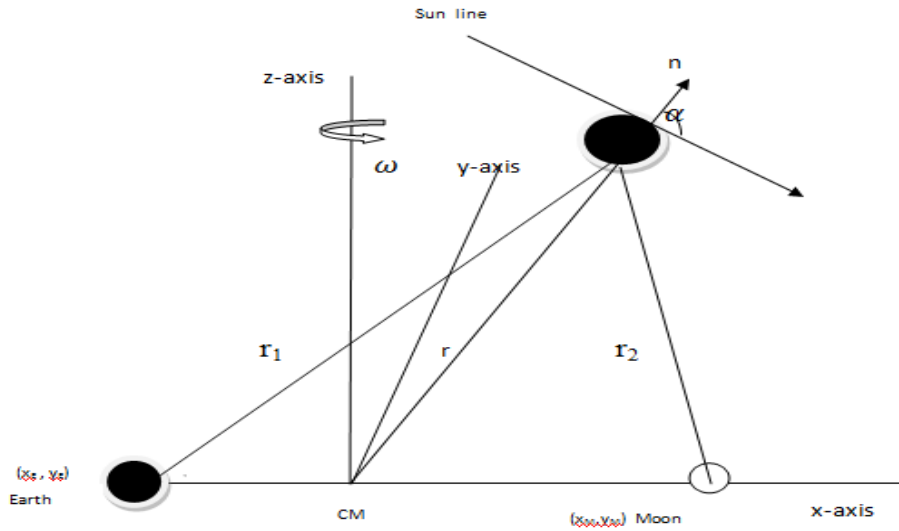


Fig. 1. The geometric of the problem

3 Equations of Motion

The vector dynamical equation for the solar sail in a rotating frame about Z-axis is described by:

$$\ddot{r} = -\frac{\partial v}{\partial x} + A_{solar\ sail} \quad (1)$$

Where v is the potential function of attraction field of m_1 and m_2

Then,

$$\ddot{x} = -\frac{\partial v}{\partial x} + A_x \quad (2)$$

$$\ddot{y} = -\frac{\partial v}{\partial y} + A_y \quad (3)$$

Using the Poincare force function $F = -v$, equations (2) and (3) become:

$$\ddot{x} = \frac{\partial F}{\partial x} + A_x \quad (4)$$

$$\ddot{y} = \frac{\partial F}{\partial y} + A_y \quad (5)$$

Where r is the position vector of the solar sail relative to the center of mass of the two primaries

$$A = A_0(S \cdot n)^2 n \quad (6)$$

A_0 is the magnitude of the solar radiation pressure force exerted on the solar sail. The unit normal to the sail n and the Sun line direct on are given by:

$$n = (\cos(\alpha)\cos(\omega t)\cos(\alpha)\sin(\omega t) \sin(\alpha)) \quad (7)$$

$$S = (\cos(\omega t) - \sin(\omega t) \ 0) \tag{8}$$

Where α is the pitch angle between the normal of the sail and the sun line.

If m_1 and m_2 rotate in their circular orbit with constant angular velocity

$\omega = \sqrt{1 + 3k/2}$ [9], then in time t the angle between the two coordinates system will be ωt then:

$$X = x \cos \omega t - y \sin \omega t \tag{9}$$

$$Y = x \sin \omega t - y \cos \omega t \tag{9.1}$$

From equation 9 we get:

$$\begin{aligned} X + iY &= (\cos \omega t - \sin \omega t)(x + iy) \\ &= e^{i\omega t}(x + iy) \end{aligned}$$

$$Z = e^{i\omega t} z \tag{10}$$

From equations (4) and (5) we obtain:

$$\ddot{X} + i\ddot{Y} = \frac{\partial F}{\partial x} + i \frac{\partial F}{\partial y} + (A_x + iA_y) \tag{11}$$

In the same way we get:

$$\frac{\partial F}{\partial x} + i \frac{\partial F}{\partial y} = \left(\frac{\partial F}{\partial x} + i \frac{\partial F}{\partial y} \right) e^{i\omega t}$$

$$(A_x + iA_y) = (A_x + iA_y) e^{i\omega t}$$

From equation (10)

$$Z = X + iY = e^{i\omega t} (x + iy) \tag{12}$$

Differentiate equation (12) with respect to t twice and substitute in equation (11) we get:

$$-\omega^2(x + iy) + 2i\omega(\dot{x} + i\dot{y}) + (\ddot{x} + i\ddot{y}) = \frac{\partial F}{\partial x} + i \frac{\partial F}{\partial y} + A_x + iA_y \tag{13}$$

Then equating the real and imaginary parts

$$\ddot{x} - \omega^2 x - 2\omega \dot{y} = \frac{\partial F}{\partial x} + A_x \tag{14.1}$$

$$\ddot{y} - \omega^2 y + 2\omega \dot{x} = \frac{\partial F}{\partial y} + A_y \tag{14.2}$$

Before attempting to solve the differential equation system (14), all quantities must be in dimensionless coordinates. To do this let $\bar{m}_1 \bar{m}_2 = l$ and $m_1 + m_2 = M$, $\frac{x}{l} = \xi$, $\frac{y}{l} = \eta$ are dimensionless coordinates of the body, and $m_3, \frac{m_1}{M} = \mu, \frac{m_2}{M} = 1 - \mu$ are dimensionless masses of the bodies m_2 and m_1 ,

Then the nondimensional equations of the system are obtained:

$$\ddot{\xi} - 2\dot{\eta} - U_x \xi = A_\xi \tag{15.1}$$

$$\ddot{\eta} + 2\dot{\xi} - U_y \eta = A_\eta \tag{15.2}$$

$$\ddot{\zeta} - U_z \zeta = A_\zeta \tag{15.3}$$

Where

$$U = \left(\frac{1}{2l^2} (x^2 + y^2) + \frac{F}{n^2 l^2} \right) \tag{16}$$

Then the potential under the effect of oblateness is given by:

$$U = \left(\frac{1}{2} (\xi^2 + \eta^2) + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} + \frac{k(1-\mu)}{\rho_1^3} \right) \tag{17}$$

Where $k = \frac{R_E^2 - R_P^2}{5R_D^2}$ is the oblateness coefficient of the larger primary, $R_E = 6378.1$ and $R_P = 6356.8$ are the equatorial and polar radii of the large primary, and $R_D = 384400$ Km is the distance between the two primaries, and

$$\frac{r_1}{l} = \rho_1, \frac{r_2}{l} = \rho_2 \text{ are dimensionless of } r_1, r_2$$

The solar sail acceleration components are obtained from equations (6), (7), and (8) in the form

$$A_\xi = a_0 \cos(\omega t) \cos^3 \alpha \tag{18.1}$$

$$A_\eta = -a_0 \sin(\omega t) \cos^3 \alpha \tag{18.2}$$

$$A_\zeta = a_0 \cos^2 \alpha \sin \alpha \tag{18.3}$$

Where the partial derivatives of the gravitational potential evaluated at the collinear libration points are: U_x, U_y and U_z

We choose a particular solution in the plane as reference to Farquhar [10] and Simo [11].

4. Laplace Transformations

Equations (15.1) and (15.2) can be solved by Laplace transforms, by putting

$$L[\ddot{\xi}] - 2L[\dot{\eta}] - L[U_x] = L[A_\xi] \tag{19}$$

$$L[\ddot{\eta}] + 2L[\dot{\xi}] - L[U_y] = L[A_\eta] \tag{20}$$

Equations (15.1) and (15.2) yield to

$$s^2 \xi(s) - s\xi(0) - \dot{\xi}(0) - 2s \eta + 2\eta(0) = \frac{1}{s} (U_\xi + A_\xi) \tag{21}$$

$$s^2 \eta(s) - s\eta(0) - \dot{\eta}(0) + 2s \xi - 2\xi(0) = \frac{1}{s} (U_\eta + A_\eta) \tag{22}$$

Using Mathematica to solve this system of equations (15.1) and (15.2), it is more convenient to consider $\dot{\xi}(0) = \dot{\eta}(0) = \dot{\zeta}(0) = 0$, $\xi(0) = 0.8$, $\eta(0) = \frac{\sqrt{3}}{2}$,

$k = 3.67146 \times 10^{-7}$ is the oblateness coefficient, $\alpha = 30^\circ$, $\omega = 1$, $a_0 = 0.2$, Then the solution will be:

$$\xi(s) = -\frac{-2a\eta - a\zeta s - 4cs - cs^3 - 7s^4 - sUx - 2Uy}{s^2(4+s^2)} \tag{23.1}$$

$$\eta(s) = -\frac{2a\xi - a\eta s - 4fs + 14s^3 - fs^3 + 2Ux - sUy}{s^2(4+s^2)} \tag{23.2}$$

We take the inverse Laplace transforms.

$$\begin{aligned} \xi(t) = & -0.064 + 0.0002\text{Cos}[t] - 0.0004t\text{Cos}[t] + 7.\text{DiracDelta}[t] - (0.00009 + 0.00009i)(\text{Cos}[2.t] \\ & - i\text{Sin}[2.t])((37969.55 + 38320.90i) - (38320.90 + 37969.55i)\text{Cos}[4.t] \\ & + \text{Cos}[t]((0. - 1.i) + 1.\text{Cos}[4.t] + (0. + 1.i)\text{Sin}[4.t]) + (37969.55 \\ & - 38320.90i)\text{Sin}[4.t]) \end{aligned}$$

$$\begin{aligned} \eta(t) = & 1.199 + t(0.1289 - 0.0004\text{Cos}[t]) - 0.00018\text{Cos}[t] + (0.00009 + 0.00009i)(\text{Cos}[2.t] - \\ & i\text{Sin}[2.t])((-38320.90 + 37969.55i) - (37969.55 - 38320.90i)\text{Cos}[4.t] - (38320.90 + \\ & 37969.55i)\text{Sin}[4.t] + \text{Cos}[t](1. -(0. + 1.i)\text{Cos}[4.t] + 1.\text{Sin}[4.t])) \end{aligned} \tag{24}$$

5 Results and Discussion

From the results at L_4 $(0.8, \frac{\sqrt{3}}{2})$. Figs. 2, 3 and 4 show the behavior of the solar sail acceleration versus time $t \in [0,50]$ and illustrate that there is periodicity.

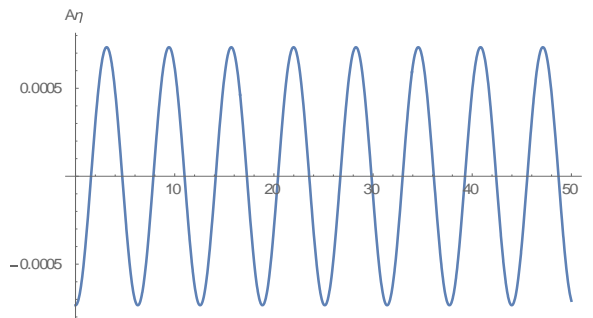


Fig. 2. The A_η acceleration derived from the solar sail about L4

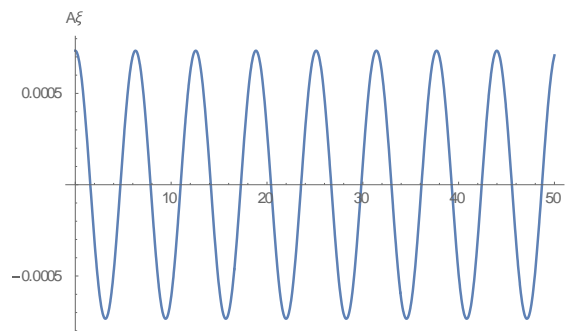


Fig. 3. The acceleration A_ζ derived from the solar sail about L4

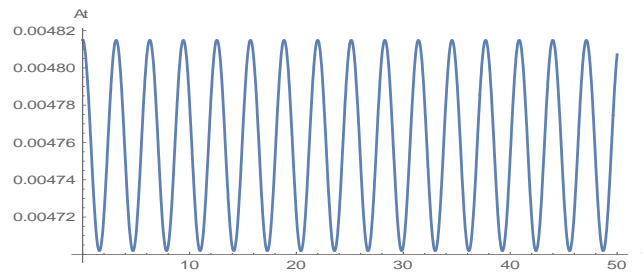


Fig. 4. The total acceleration derived from the solar sail about L4

Figs. 5 and 6 show that the motion of a spacecraft around the libration point appears at the plane of motion. See Fig. 7 which illustrates the trajectory of a spacecraft about the L4. Fig. 8 shows the phase space of a spacecraft revolves in an ellipse around the L4.

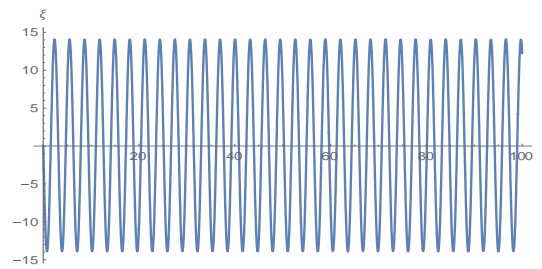


Fig. 5. Trajectory near the point L4 with ξ -axis

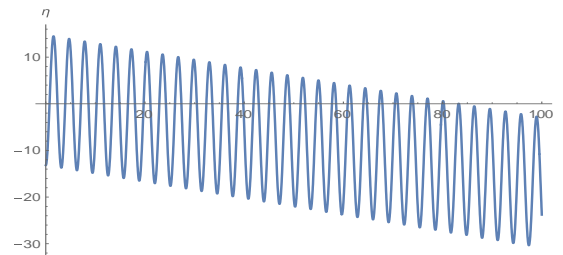


Fig. 6. Trajectory near the point L4 with η -axis

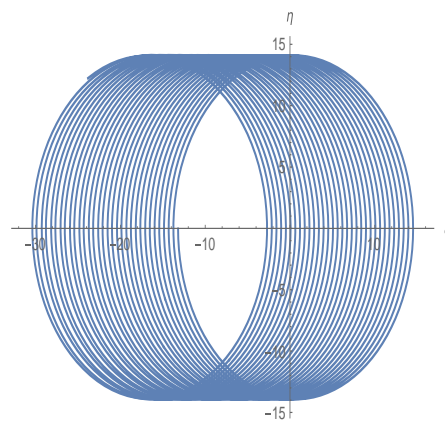


Fig. 7. Trajectory of a spacecraft about L4

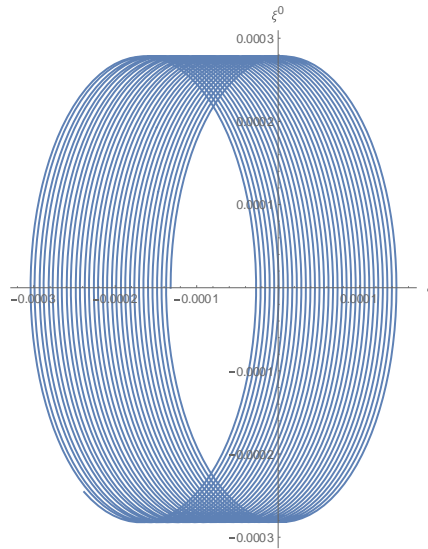


Fig. 8. The phase space of the body about L4

From the results at $L_2 (1.2, 0)$. Figs. 9, 10 and 11 show the behavior of the solar sail acceleration versus time $t \in [0,50]$, and it illustrates that there is periodicity. Figs. 12 and 13 show that the motion of a spacecraft around L_2 for $t \in [0,50]$ on the ξ -axis and then η -axis, where the effect of oblateness appears at the plane of motion, is stable and escape after $t \cong 35$. Fig. 14 illustrates the trajectory of a spacecraft about the L_2 . Fig. 15 shows the phase space of a spacecraft revolves in an ellipse around the L_2 .

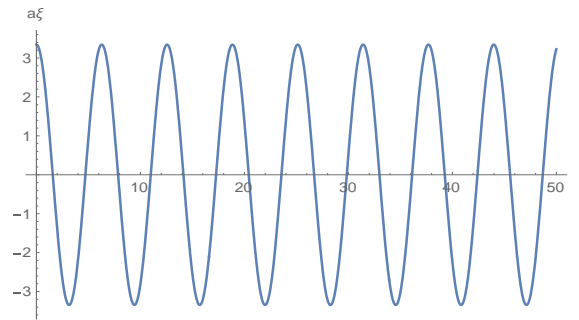


Fig. 9. The acceleration a_ξ derived from the solar sail about L2

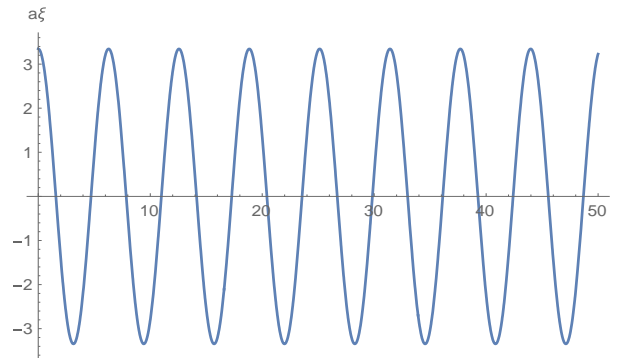


Fig. 10. The acceleration a_η derived from the solar sail about L2

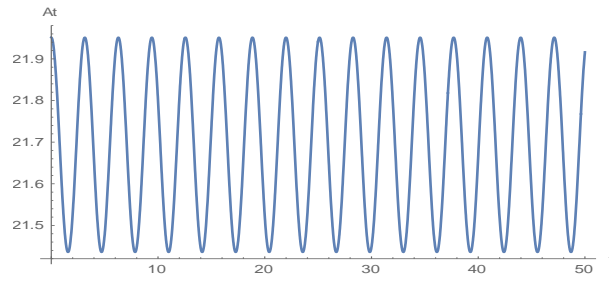


Fig. 11. The total Acceleration derived from the solar sail about L2

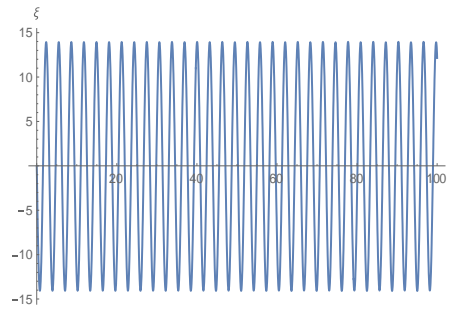


Fig. 12. Trajectory near the point L2 with ξ -axis

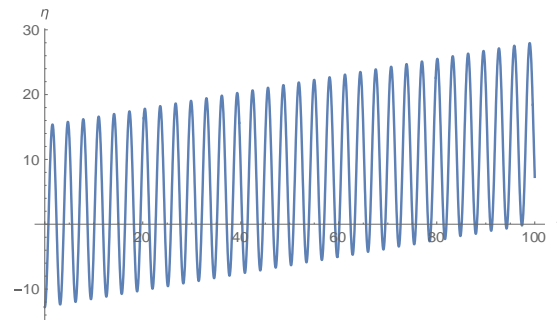


Fig. 13. Trajectory near the point L2 with η -axis

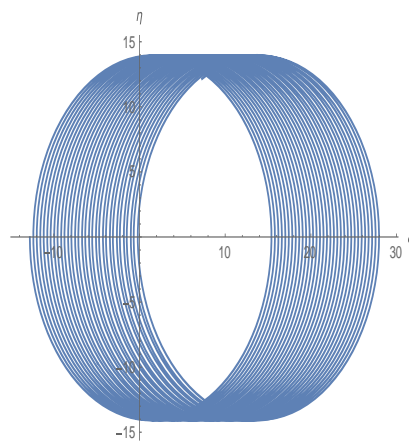


Fig. 14. Trajectory of aspacecraft about L2

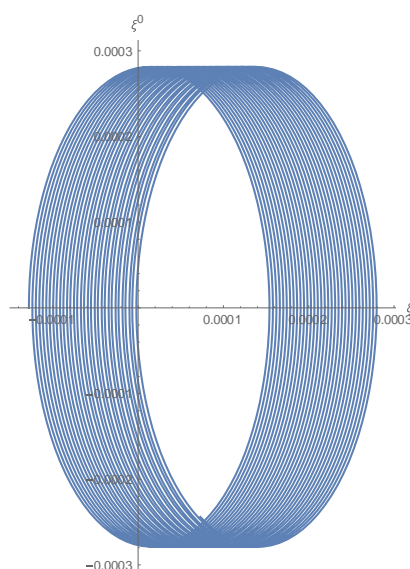


Fig. 15. The phase of the body about L2

6 Conclusion

Through this work the behavior of a body about the L2 and L4 are studied by Laplace transformation. The results obtained by Laplace transformations were very interesting to specify this study. Through this work the effect of oblateness due to the bigger primary playing a great role in the space missions. So, that the oblateness effect is considered when any missions designed. An application has done for the motion of spacecraft near the equilibrium points of the Earth-Moon system and the results obtained was in a good agreement with the previous work [12].

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Baoyin H, McInnes C. J. Guid Control Dyn. 2006;29:538-543.
- [2] Verrier P, Waters T, Sieber J. Celest. Mech. Dyn. Astron. 2014;120:373-400.
- [3] Waters TJ, McInnes CR. Periodic orbits above the ecliptic in the solar-sail restricted three-body problem. Journal of Guidance, Control and Dynamics. 2007;30(3):687-693.
- [4] McInnes CR, McDonald AJ, Simmons JF, MacDonald EW. Solar sail parking in restricted three-body systems. Journal of Guidance, Control and Dynamics. 1994;17(2):399-406.
- [5] Kumari Reena, Kushvah, Badam Singh. Equilibrium points and zero velocity surfaces in the restricted four-body problem with solar wind drag. Ap & Ss. 2013;344:2:347-359.
- [6] Jules Verne. From the earth to the moon. Revision: B1.U.S; 2008.

- [7] Song M, He X, He D. Displaced orbits for solar sail equipped with reflectance control devices in Hill's restricted three-body problem with oblateness. *Astrophysics and Space Science*. 2016;361(10):327.
- [8] Ishwar B, Kushvah BS. Linear stability of triangular equilibrium points in the generalized photogravitational restricted three body problem with pointing Robertson drag. *Journal of Dynamical Systems and Geometric Theories*. 2006;4(1):79-86.
- [9] Frank D, Stacey, Davis PM. *Physics of the Earth*. Cambridge University Press; 2008.
- [10] Farquhar R. The utilization of halo orbits in advanced lunar operations. Nasa Technical Report; 1971.
- [11] Simo J, Molnnes C. *Commun Nonlinear Sci*. 2009a;14:4191-4196.
- [12] Ming Song, et al. Displaced orbits for solar sail equipped with reflectance control devices in Hill's restricted three body problem with oblateness. 2016;361:327.

© 2020 Elmalky et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<http://www.sdiarticle4.com/review-history/59375>