nternational Journal of Management and Humanities (IJMH)
ISSN: 2394-0913 (Online), Volume-4 Issue-8, April 2020

# Vertex Polynomial of Middle, Line and Total Graphs of some standard Graphs

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***Abstract:*** ***In this article, we have obtained vertex polynomial of Middle, Line and Total graphs of some standard graphs namely complete graph, star, path, cycle and wheel graph.***

***Keywords*****:** ***Vertex polynomial, Middle graph, Line Graph, Total Graph***

## I.INTRODUCTION

In this paper, we have considered only finite graphs which are non-trivial, undirected, having no self- loops and having no parallel edges. Two edges of a graph are adjacent if they have a common vertex. If a vertex $v$ is an end vertex of an edge $e$, then edge $e$ is incident on vertex $v$. Vertices and edges are called elements of a graph and are neighbors if they are either incident or adjacent. $K_{1,n}$ represents a star. A wheel graph $W_n$ and is defined as $W_n = K_1 + C_{n-1.}$ A graph $G$ is $k-$ regular if each vertex has degree $k$. If $x = uv$ is an edge of a graph $G$, and $w$ is not a vertex of $G$, then $x$ is said to subdivided if it is replaced by the edges $uw$ and $vw$. For terminologies and notations, we refer [1]. The vertex polynomial of the graph G [2] is defined as $V(G;x) = \sum_{k=0}^{\triangle(G)} v_k x^k$, where $\triangle(G) = \max\{d(v): v \in V(G)\}$ and $v_k$ is the number of vertices of degree $k$. The roots of a vertex polynomial are called vertex polynomial roots.

## II.MIDDLE GRAPH, LINE GRAPH, TOTAL GRAPH AND VERTEX POLYNOMIAL WITH AN EXAMPLE.

In this part, we list the definitions of Middle graph, Line graph and Total graph of a graph with an example. Also we show their vertex polynomials.

Let $G$ be a graph with $|V| = m$ and $|E| = n$. Its line graph $L(G)$ contains vertices which are edges of $G$ and edges in $L(G)$ are drawn between vertices if they have common vertex in $G$. Middle graph $M(G)$ has vertex set containing all vertices and all edges of $G$ and edges of $M(G)$ between new vertices are drawn only when the edges corresponding in $G$ are adjacent. Total graph $T(G)$ has vertex set containing all vertices of $G$ and all edges of $G$ and two vertices of total graph are adjacent if they happen to be neighbors in $G$. The following figures show a graph, its line graph, middle graph and total graph with their vertex polynomials.

Revised Manuscript Received on April 03, 2020.
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**Figure 1− Graph $G$.**

In this graph, we find 1, 2 and 1 vertices of degrees 1, 2 and 3 respectively. Thus $V(G;x) = x + 2x^2 + x^3$ . Now we consider its line graph as in Figure 2.

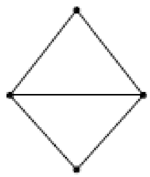

**Figure 2−$L(G)$**

We note that $L(G)$ has four vertices of which two are of degree 2 and remaining two are of degree 3. Thus
$V(L(G);x) = 2x^2 + 2x^3$
Now, we consider middle graph $M(G)$ as shown in Figure 3.

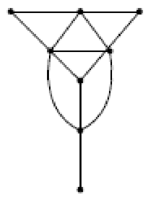

**Figure 3−$M(G)$**

The middle graph has one vertex of degree 1, two vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and two vertices of degree 5. Thus $V(M(G);x) = x + 2x^2 + x^3 + 2x^4 + 2x^5$. Now we consider total graph $T(G)$ as shown in Figure 4.

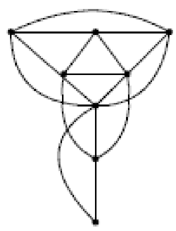

**Figure 4−$T(G)$**

$T(G)$, the total graph has1, 4, 2 and 1 vertices of degrees 2, 4, 5 and 6 respectively.

Thus $V(T(G);x) = x^2 + 4x^4 + 2x^5 + x^6$

## III.MAIN RESULTS

**Theorem 3.1:** If $M(K_n)$ is middle graph of $K_n$, then

$$V(M(K_n);x) = nx^{n-1} + \frac{n(n-1)}{2}x^{2n-2}$$

**Proof:** We note that $K_n$ has $n$ vertices, whereas its middle graph $M(K_n)$ has $\frac{n^2+n}{2}$ vertices by definition of middle graph. We can partition vertex set of $M(K_n)$ into two disjoint subsets $V_1$ and $V_2$ as follows:

$V_1$: Vertex set with $|V_1| = n$, degree being $(n-1)$ for each vertex and

$V_2$: Vertex set with$|V_2| = \frac{n^2-n}{2}$, degree being $(2n-2)$ for each vertex.

Hence $V(M(K_n);x) = nx^{n-1} + \frac{n(n-1)}{2}x^{2n-2}$

**Theorem 3.2:** If $M(K_{1,n})$ is middle graph of $K_{1,n}$, then $V(M(K_{1,n});x) = nx^{n+1} + x^n + nx$ for $n \geq 2$

**Proof:** We note that $K_{1,n}$ has $(n+1)$ vertices, whereas its middle graph $M(K_{1,n})$ has $(2n+1)$ vertices by definition of middle graph. We can partition the vertex set of $M(K_{1,n})$ into disjoint subsets $V_1$ $V_2$and $V_3$ as follows:

$V_1$: Vertex set containing 1 vertex of degree n,

$V_2$: Vertex set with $|V_2| = n$, each vertex being pendant and

$V_3$: Vertex set $|V_3| = n$,degree being $(n+1)$ for each vertex.

Thus $V(M(K_{1,n});x) = nx^{n+1} + x^n + nx$

**Theorem 3.3:** If $M(P_n)$ is the middle graph of $P_n$, then

$$V(M(P_n);x) = 2x + (n-2)x^2 + (n-1)x^3, n \geq 3$$

**Proof:** We note, by definition of middle graph that, $M(P_n)$ contains $(2n-1)$vertices. We can partition the vertex set of $M(P_n)$ into three disjoint subsets $V_1$ $V_2$and $V_3$ as follows:

$V_1$: Vertex set containing 2 vertices, each of degree 1,

$V_2$: Vertex set with $|V_2| = n-2$, degree being 2 for each vertex and

$V_3$: Vertex set with $|V_3| = n-1$ , degree being 3 for each vertex.

Thus $V(M(P_n);x) = 2x + (n-2)x^2 + (n-1)x^3$

**Theorem 3.4:** If $M(C_n)$ is the middle graph of $C_n$, then

$$V(M(C_n);x) = nx^2 + nx^4$$

**Proof:** We note, by definition of middle graph that, $M(C_n)$ contains $2n$ vertices. We can partition the vertex set of $M(C_n)$ into $V_1$and $V_2$ as follows:

$V_1$: Vertex set containing $n$ vertices of $C_n$ , each of degree 2 and

$V_2$: Vertex set containing $n$ new vertices obtained by definition of middle graph, each of degree 4.

Thus $V(M(C_n);x) = nx^2 + nx^4$

**Corollary 3.5:** The vertex polynomial roots of $M(C_n)$ are 0, 0 and $\pm i$

**Proof:** In view of Theorem 3.4, vertex polynomial roots of $M(C_n)$ can be obtained by taking

$$nx^2 + nx^4 = 0 \quad \Rightarrow \quad x = 0,0 \text{ and } \pm i$$

**Theorem 3.6:** If $M(W_n)$ is the middle graph of $W_n$, then

$$V(M(W_n);x) = (n-1)x^{n+2} + x^{n-1} + (n-1)x^6 + (n-1)x^3, \text{ for } n \geq 5$$

**Proof:** We note, by definition of middle graph that, $M(W_n)$ contains $(3n-2)$ vertices. We can partition vertex set of $M(W_n)$ into four disjoint subsets $V_1, V_2, V_3$ and $V_4$ as follows:

$V_1$: Vertex set containing a single vertex of degree $(n-1)$,

$V_2$: Vertex set with $|V_2| = n-1$, degree being 3 for each vertex,

$V_3$: Vertex set $|V_3| = n-1$ , degree being 6 for each vertex, and

$V_4$: Vertex set $|V_4| = n-1$ , degree being $n+2$ for each vertex.

Thus $V(M(W_n);x) = (n-1)x^{n+2} + x^{n-1} + (n-1)x^6 + (n-1)x^3$

**Theorem 3.7:** If $L(K_n)$ is the line graph of $K_n$ ,then

$$V(L(K_n);x) = \frac{n(n-1)}{2}x^n$$

**Proof:** Using definition of line graph, we easily understand that $L(K_n)$ is a $n-$ regular graph with $\frac{n^2-n}{2}$ vertices and every vertex will be of degree $n$.

Thus $V(L(K_n;x)) = \frac{n(n-1)}{2}x^n$.

**Theorem 3. 8:** If $L(K_{1,n})$ is the line graph of $K_{1,n}$, then $V\left(L(K_{1,n});x\right) = nx^{n-1}$

**Proof:** By definition of line graph, of a graph, we easily understand that $L(K_{1,n})$ is a complete graph $K_n$and therefore every vertex will be of degree$(n-1)$.

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DOI: 10.35940/ijmh.H0794.044820
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Thus $V\left(L(K_{1,n};x)\right) = nx^{n-1}$.

**Theorem 3.9:** If $L(P_n)$ is the line graph of $P_n$, then $V\left(L(P_n);x)\right) = 2x + (n-3)x^2, n \geq 3$

**Proof:** By definition of line graph, we easily understand that $L(P_n)$ is a path graph $P_{n-1}$.
Thus $V\left(L(P_n);x)\right) = V(P_{n-1};x)$.
It is easy to note that path graph $P_{n-1}$ will have 2 pendant vertices and remaining $(n-3)$ internal vertices will be of degree 2.
Thus $V\left(L(P_n);x)\right) = 2x + (n-3)x^2$.

**Theorem 3.10:** If $L(C_n)$is the line graph of $C_n$, then is $V\left(L(C_n)\,;x)\right) = nx^2$.

**Proof:** By definition of line graph , we easily understand that $L(C_n) = C_n$ itself. As degree of every vertex of $C_n$ is 2 and as it has $n$ vertices, we find $V\left(L(C_n)\,;x)\right) = nx^2$.

**Corollary 3.11:** The vertex polynomial root of $L(C_n)$ is 0 with multiplicity 2.

**Proof:** In view of Theorem 3.10, vertex polynomial roots of $L(C_n)$ can be obtained by taking

$$nx^2 = 0 \quad \Rightarrow \quad x = 0,0 \text{ as } n \neq 0$$

**Theorem 3.12:** If $L(W_n)$ is the line graph of $W_n$, then

$$V\left(L(W_n\,;x)\right) = (n-1)x^4 + (n-1)x^n, \qquad n \geq 4$$

**Proof:** We know, a wheel graph $W_n$ will have $n$ vertices and $(2n-2)$edges. The vertex set of $L(W_n)$ will have $(2n-2)$ vertices and we can partition the vertex set into two disjoint subsets $V_1$ and $V_2$ as follows:

$V_1$: Vertex set with $|V_1| = n-1$, degree being 4 for each vertex

$V_2$: Vertex set with $|V_2| = n-1$, degree being $n$ for each vertex

Thus $V\left(L(W_n\,;x)\right) = (n-1)x^4 + (n-1)x^n$

**Theorem 3.13: If** $T(K_n)$ is the total graph of $K_n$, then

$$V(T(K_n);x) = \frac{n(n+1)}{2}x^{2n-2}$$

**Proof:** We note that $K_n$will have $n$ vertices and $\frac{n(n-1)}{2}$ edges. By definition of total graph of a graph, we note that, total graph $T(K_n)$ will have $n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$ vertices and each vertex will be of degree $(2n-2)$.

Thus

$$V(T(K_n);x) = \frac{n(n+1)}{2}x^{2n-2}$$

**Theorem 3.14:** If $T(K_{1,n})$ is the total graph of $K_{1,n}$, then

$$V(T\left(K_{1,n};x\right) = nx^2 + nx^{n+1} + x^{2n}$$

**Proof:** We know a star graph $K_{1,n}$will have $n+1$ vertices and $n$ edges. We note that total graph $T(K_{1,n})$ of star graph $K_{1,n}$will have $(2n+1)$ by the way in which total graph is defined and we can partition this the vertex set into 3 disjoint subsets $V_1$, $V_2$and $V_3$ as follows:

$V_1$: Vertex set containing a single vertex of degree $2n$,

$V_2$: Vertex set with $|V_2| = n-1$, degree being 2 for each vertex and

$V_3$: Vertex set $|V_3| = n$ , degree being 2 for each vertex.
Thus $V(T\left(K_{1,n};x\right) = nx^2 + nx^{n+1} + x^{2n}$

**Theorem 3.15:** If $T(P_n)$ is the total graph of $T(P_n)$, then

$$V(T(P_n);x) = 2x^2 + 2x^3 + (2n-5)x^4, n \geq 3$$

**Proof:** We know, a path graph $P_n$ will have $n$ vertices and $(n-1)$edges. By definition of total graph, we note h $T(P_n)$ will have $(2n-1)$ vertices and we can partition this vertex set into 3 disjoint subsets $V_1$, $V_2$and $V_3$ as follows:

$V_1$: Vertex set containing 2 vertices, each of degree 2,

$V_2$: Vertex set containing 2 vertices, each of degree 3 and

$V_3$: Vertex set containing $(2n-5)$ vertices, each of degree 4.

Thus $V(T(P_n);x) = 2x^2 + 2x^3 + (2n-5)x^4$

**Theorem 3.16:** If $T(C_n)$ is the total graph of $C_n$, then

$$V(T(C_n);x) = 2nx^4$$

**Proof:** We know, a cycle graph $C_n$ has $n$ vertices. By definition of total graph of a graph, we note that, $T(C_n)$ of $C_n$ is a 4− regular graph with $2n$ vertices and each vertex will be of degree 4. Thus $V(T(C_n);x) = 2nx^4$.

**Corollary 3.17:** The vertex polynomial root of $T(C_n)$ is 0 with multiplicity 4.

**Proof:** In view of Theorem 3.16, result is obvious.

**Theorem 3.18:** If $T(W_n)$ is the total graph of $W_n$, then

$$V(T(W_n;x) = (2n-2)x^6 + (n-1)x^{n+2} + x^{2n-2}, \text{ for } n \geq 5$$

**Proof:** We know a wheel graph $W_n$ has $n$ vertices and $2n-2$ edges. By definition of total graph, we note that $T(W_n)$ of $W_n$ will have $(3n-2)$ vertices and we can partition this vertex set into three disjoint subsets $V_1$, $V_2$ and $V_3$ as follows:

$V_1$: Vertex set containing $(2n-2)$ vertices, each of degree 6,

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$V_2$: Vertex set containing a single vertex of degree $(2n-2)$ and

$V_3$: Vertex set containing $(n-1)$ vertices, each of degree $(n+2)$.

Thus $V(T(W_n;x) = (2n-2)x^6 + (n-1)x^{n+2} + x^{2n-2}$

## IV.CONCLUSION

Vertex polynomial is one of the possible ways in which we can represent a graph algebraically. We can compare this polynomials with other polynomials associated with graph also.

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