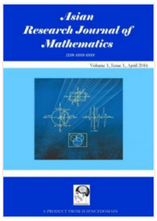

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# Numerical Solution of Voltera Integro-Differential Equations of Type 2 Using Adomian Decomposition Method

**Joseph Bonazebi Yindoula[1*], Alphonse Massamba[1] and Gabriel Bissanga[1]**

[1] *Laboratory of Numerical Analysis, Computer Science and Applications, Marien Ngouabi Université, Brazzaville, Congo.*

***Authors' contributions***

*This work was carried out in collaboration between all authors. Author JBY designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author AM managed the analyses of the study. Author GB managed the literature searches. All authors read and approved the final manuscript.*

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## Abstract

In this paper, we present a numerical method for solving linear and nonlinear Voltera integral-differential equations.

**Corresponding author: E-mail: bonayindoula@yahoo.fr;*

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# 1 Introduction

Many phenomena of the physical world were described by integro-differential equations [1]. These equations play a great rôle in the development in the explanation of new physical phenomena and in the mathematcal formulation of several theories. For example, in the study of continuous media or fields one is most often,interest in one of the following classes of phenomena:

(1) phenomena which consist of the propagation of some action; the medium itself is not transported from one place to another; typical is the propagation of sound waves in turbulent medium itself is not transported from one place to another[2];
(2) phenomena in which there is transport or diffusion of biological molecules in porous soils [3].

Each class is ruled by essentialty one type integro-differential equation. Nowadays, integro-differential equations arise in many physical processes, such as glass forwing proces[4] . nanohydrodynamics [5] , drop wise condensation [6] , and wind ripple in the desert [7] . There are various numerical and analytical methods to solve such problems, but each method limits to a special class of integro-differential equations.
The purpose of this paper is to extend the analyses of Adomian decomposition method to solve unidimensional example of Voltera linear and non linear integro-differential equations. General for of k kind no linear integro-differential equations is :

$$\begin{cases} u^{(k)}(x) = f(x) + \lambda \int\limits_0^x K(x,t)N(t,u(t))dt \\ u^{(i)}(0) = a_i, \quad 0 \leq i \leq k-1 \end{cases} \tag{1}$$

where $u^{(k)}(0) = \frac{d^k}{dx^k}(u(0))$ .

In this paper, we present another numerical approach based on Adomian decomposition method for solving second kind Voltera integro-differential equations.

This method is the combination of the method of the constants and the decompsition method of Adomian. However, operator function in Voltera equation is'nt constant and we note that H(x). Consequently, the new Voltera equation can obtain canonic form and Algorithm of Adomian.

# 2 Principle of Method

By $k$ degree integration of equation (1), we obtain:

$$u(x) = F(x) + \lambda \int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)N(t,u(t))dtdx_1 \cdots dx_k \tag{2}$$

with

$$F(x) = \sum_{i=0}^{k} a_i \frac{x^i}{i!} + \int_0^x \int_0^{x_1} \cdots \int_0^{x_{k-1}} f(x_k) dx_1 \cdots dx_k \tag{3}$$

Let's make

$$H(x) = \lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) N(t, u(t)) dt dx_1 \cdots dx_k \tag{4}$$

Then from equation (2) we have

$$u(x) = F(x) + H(x) \tag{5}$$

By replacing $u(t)$ by this expression in $H$ form, we have canonic form of Adomian:

$$\left\{ \begin{array}{l} H(x) = \lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) N(t, u(t) dt dx_1 \cdots dx_k \\ \qquad +\lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) \left[ N(t, F(t) + H(t)) - N(t, u(t) \right] dt dx_1 \cdots dx_k \end{array} \right. \tag{6}$$

Thus, we obtain Adomian algorithm

$$\left\{ \begin{array}{l} H_0(x) = \lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) N(t, u(t) dt dx_1 \cdots dx_k \\ H_{n+1}(x) = \ +\lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) \left[ N(t, F(t) + H_n(t)) - N(t, u(t) \right] dt dx_1 \cdots dx_k, \forall n \geq 0 \end{array} \right. \tag{7}$$

Thus,$H(x)$ is obtain by the formula:

$$H(x) = \sum_{n=0}^{+\infty} H_n(x) \tag{8}$$

We can determine $H(x)$ and solution of the equation (2) is

$$u(x) = F(x) + H(x) \tag{9}$$

# 3 Convergence of New Approach

## 3.1 Proposition

$\forall x \in [a, \varepsilon], |F(x)| \leq m, \forall (x, t) \in [a, \varepsilon]^2, |K(x, t)| \leq M, |N(t, u(t)| \leq \varepsilon$ and $N(t, u(t))$ satisfy the condition $|N(t, y_1) - N(t, y_2)| \leq L |y_1 - y_2|$

Then, the following algorithm is converging

$$\left\{ \begin{array}{l} H_0(x) = \lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) N(t, u(t) dt dx_1 \cdots dx_k \\ H_{n+1}(x) = \ \lambda \int_0^x \int_0^{x_1} \cdots \int_0^{x_k} K(x_k, t) \left[ N(t, F(t) + H_n(t)) - N(t, u(t) \right] dt dx_1 \cdots dx_k, \forall n \geq 0 \end{array} \right. \tag{10}$$

## 3.2 Proof

We know

$$\begin{cases} |H_0(x)| = \left|\lambda \int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)N(t,u(t)dtdx_1 \cdots dx_k\right| \\ |H_{n+1}(x)| = \left| \lambda \int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)\left[N(t,F(t)+H_n(t)) - N(t,u(t)\right] dtdx_1 \cdots dx_k \right|, \forall n \geq 0 \end{cases} \tag{11}$$

$\Longleftrightarrow$

$$\begin{cases} |H_0(x)| = |\lambda| \left|\int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)N(t,u(t)dtdx_1 \cdots dx_k\right| \leq |\lambda| M\varepsilon(x-a) \\ |H_1(x)| = |\lambda| \left| \int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)\left[N(t,F(t)+H_0(t)) - N(t,u(t)\right] dtdx_1 \cdots dx_k \right| \\ \qquad \leq \frac{(|\lambda| M\varepsilon(x-a))^2}{2!} \varepsilon L \\ \vdots \\ |H_n(x)| = |\lambda| \left| \int\limits_0^x \int\limits_0^{x_1} \cdots \int\limits_0^{x_k} K(x_k,t)\left[N(t,F(t)+H_{n-1}(t)) - N(t,u(t)\right] dtdx_1 \cdots dx_k \right| \\ \qquad \leq \frac{(|\lambda| M\varepsilon(x-a))^{n+1}}{(n+1)!} (\varepsilon L)^n \end{cases} \tag{12}$$

hence from (12) we have

$$\sum_{n=0}^{+\infty} |H_n(x)| \leq \sum_{n=0}^{+\infty} \frac{(|\lambda| M\varepsilon(x-a))^{n+1}}{(n+1)!} (\varepsilon L)^n = \sum_{i=0}^{+\infty} \frac{(|\lambda| M\varepsilon L(x-a))^i}{i!} \frac{1}{\varepsilon L} = \frac{\exp(|\lambda| M\varepsilon L(x-a)) - 1}{\varepsilon L} \tag{13}$$

So the method is convergent

# 4 Application

## 4.1 Second kind voltera intogro-differential equations

### 4.1.1 Example 1

Let us consider second kind Voltera linear integro-differential equation :

$$\begin{cases} u'(x) = 1 - \int\limits_0^x u(t)dt \\ u(0) = 1 \end{cases} \tag{14}$$

**Solving**

Let us for state equation

$$u'(x) = 1 - \int_0^x u(t)dt \tag{15}$$

By integration equation (15) so $x$, we have canonic form of Adomian:

$$u(x) = 1 + x - \int_0^x \left( \int_0^z u(t)dt \right) dz \tag{16}$$

Let's make

$$H(x) = \int_0^x \left( \int_0^z u(t)dt \right) dz \tag{17}$$

the equation (16) is written

$$u(x) = 1 + x - H(x) \tag{18}$$

By replacing $u(x)$ of equation (17), we obtain:

$$H(x) = \int_0^x \left( \int_0^z (1 + t - H(t))\, dt \right) dz \tag{19}$$

Thus

$$H(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 - \int_0^x \left( \int_0^z H(t)dt \right) dz \tag{20}$$

Let's make

$$H(x) = \sum_{n=0}^{+\infty} H_n(x) \tag{21}$$

we have

$$\sum_{n=0}^{+\infty} H_n(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 - \sum_{n=0}^{+\infty} \int_0^x \left( \int_0^z H_n(t)dt \right) dz \tag{22}$$

to find Adomian algorithm:

$$\begin{cases} H_0(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 \\ H_{n+1}(x) = -\int_0^x \left( \int_0^z H_n(t)dt \right) dz, \forall n \geq 0 \end{cases} \tag{23}$$

After calculations, this algorithm gives

$$\begin{cases} H_0(x) = \frac{1}{3!}x^3 + \frac{1}{2!}x^2 \\ H_1(x) = -\frac{1}{5!}x^5 - \frac{1}{4!}x^4 \\ H_2(x) = \frac{1}{7!}x^7 + \frac{1}{6!}x^6 \\ H_3(x) = -\frac{1}{9!}x^9 - \frac{1}{8!}x^8 \\ \vdots \end{cases} \tag{24}$$

So, $H(x)$ can be written as:

$$\begin{cases} H(x) = H_0(x) + H_1(x) + H_2(x) + H_3(x) + \cdots \\ \quad = \frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{120}x^5 - \frac{5}{120}x^4 + \frac{1}{5040}x^7 + \frac{1}{720}x^6 - \frac{1}{362\,880}x^9 - \frac{1}{40\,320}x^8 + \cdots \\ \quad = \left(\frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{1}{720}x^6 - \frac{1}{40\,320}x^8 + \cdots\right) + \left(\frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{5040}x^7 - \frac{1}{362\,880}x^9 + \cdots\right) \end{cases} \tag{25}$$

Consequently

$$\begin{cases} u(x) = 1 + x - H(x) \\ u(x) = 1 + x - \left(\frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{1}{720}x^6 - \frac{1}{40\,320}x^8 + \cdots\right) - \left(\frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{5040}x^7 - \frac{1}{362\,880}x^9 + \cdots\right) \\ u(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40\,320}x^8 - \cdots\right) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362\,880}x^9 + \cdots\right) \\ u(x) = \cos x + \sin x \end{cases} \tag{26}$$

With exact solution equation of example 1:

$$u(x) = \cos x + \sin x \tag{27}$$

### 4.1.2 Example 2

Consider the following integro-differential equation:

$$\begin{cases} u'(x) = e^x\left(1 - xe^x\right) + \int_0^x e^{2x-t}u(t)dt \\ u(0) = 1 \end{cases} \tag{28}$$

**Solving**

Let us consider the following equation

$$u'(x) = e^{x}\left(1 - xe^{x}\right) + \int_0^x e^{2x-t}u(t)dt \tag{29}$$

Integration of equation (26) gives canonic form of Adomian:

$$u(x) = e^{x} - \frac{1}{4} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + \int_0^x \left(\int_0^s e^{2s-t}u(t)dt\right) ds \tag{30}$$

Let's make

$$H(x) = \int_0^x \left(\int_0^s e^{2s-t}u(t)dt\right) ds \tag{31}$$

Equation (30) gives:

$$u(x) = e^{x} - \frac{1}{4} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + H(x) \tag{32}$$

By replacing equation (32), equation (31) gives:

$$H(x) = \int_0^x \left(\int_0^s e^{2s-t}\left(e^{t} - \frac{1}{4} - \frac{1}{2}te^{2t} + \frac{1}{4}e^{2t}\right) dt\right) ds + \int_0^x \left(\int_0^s e^{2s-t}H(t)dt\right) ds \tag{33}$$

Then

$$H(x) = \left(\frac{1}{4} + \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) + \frac{1}{4}e^{x} - \frac{1}{2}e^{2x} + \frac{11}{36}e^{3x} - \frac{1}{6}xe^{3x} - \frac{1}{18} + \int_0^x \left(\int_0^s e^{2s-t}H(t)dt\right) ds \tag{34}$$

Equation (34) is canonic form of the new approach: Let's make:

$$H(x) = \sum_{n=0}^{+\infty} H_n(x) \tag{35}$$

By replacing equation (35) in equation (34), we have:

$$\sum_{n=0}^{+\infty} H_n(x) = \left(\frac{1}{4} + \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) + \frac{1}{4}e^{x} - \frac{1}{2}e^{2x} + \frac{11}{36}e^{3x} - \frac{1}{6}xe^{3x} - \frac{1}{18} + \sum_{n=0}^{+\infty}\int_0^x \left(\int_0^s e^{2s-t}H_n(t)dt\right) ds \tag{36}$$

Then, algorithm Adomian modified is :

$$\begin{cases} H_0(x) = \frac{1}{4} + \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} \\ H_1(x) = \frac{1}{4}e^{x} - \frac{1}{2}e^{2x} + \frac{11}{36}e^{3x} - \frac{1}{6}xe^{3x} - \frac{1}{18} + \int\limits_0^x \left( \int\limits_0^s e^{2s-t}H_0(t)dt \right) ds \\ H_{n+1}(x) = \int\limits_0^x \left( \int\limits_0^s e^{2s-t}H_n(t)dt \right) ds \end{cases} \tag{37}$$

Calculate $H_1(x)$

$$\begin{cases} H_1(x) = \frac{1}{4}e^{x} - \frac{1}{2}e^{2x} + \frac{11}{36}e^{3x} - \frac{1}{6}xe^{3x} - \frac{1}{18} + \int\limits_0^x \left( \int\limits_0^s e^{2s-t}H_0(t)dt \right) ds \\ \quad = \frac{1}{4}e^{x} - \frac{1}{2}e^{2x} + \frac{11}{36}e^{3x} - \frac{1}{6}xe^{3x} - \frac{1}{18} + \frac{1}{6}xe^{3x} - \frac{1}{4}e^{x} + \frac{1}{2}e^{2x} - \frac{11}{36}e^{3x} + \frac{1}{18} \\ H_1(x) = 0 \end{cases} \tag{38}$$

Recursive calculate gives

$$H_n(x) = 0; \forall n \geq 1 \tag{39}$$

Consequently

$$H(x) = \sum_{n=0}^{+\infty} H_n(x) = H_0(x) = \frac{1}{4} + \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} \tag{40}$$

Then, we have following exact solution of example 2:

$$\begin{cases} u(x) = e^{x} - \frac{1}{4} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + H(x) \\ \quad = e^{x} - \frac{1}{4} + \frac{1}{4} - \frac{1}{2}xe^{2x} + \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} - \frac{1}{4}e^{2x} \\ \quad = e^{x} \end{cases} \tag{41}$$

Then

$$u(x) = e^{x} \tag{42}$$

## 4.2 Type 2 Voltera nonlinear integro-differentials equations

### 4.2.1 Example 3

Consider the following type 2 Voltera integro-differential equation :

$$\begin{cases} u'(x) = \frac{9}{4} - \frac{5}{2}x - \frac{1}{2}x^2 - 3e^{-x} - \frac{1}{4}e^{-2x} + \int\limits_0^x (x-t)u^2(t)dt \\ u(0) = 2 \end{cases} \tag{43}$$

**Solving by Adomian**

We have

$$u'(x) = \frac{9}{4} - \frac{5}{2}x - \frac{1}{2}x^2 - 3e^{-x} - \frac{1}{4}e^{-2x} + \int\limits_0^x (x-t)u^2(t)dt \tag{44}$$

Integration as $x$ of equation (41) gives

$$u(x)=u(0)+\int_0^x\left(\frac{9}{4}-\frac{5}{2}s-\frac{1}{2}s^2-3e^{-s}-\frac{1}{4}e^{-2s}\right)ds+\int_0^x\left(\int_0^s(s-t)u^2(t)dt\right)ds \tag{45}$$

Then

$$u(x)=\frac{9}{4}x+3e^{-x}+\frac{1}{8}e^{-2x}-\frac{5}{4}x^2-\frac{1}{6}x^3-\frac{9}{8}+\int_0^x\left(\int_0^s(s-t)u^2(t)dt\right)ds \tag{46}$$

Let's make:

$$H(x)=\int_0^x\left(\int_0^s(s-t)u^2(t)dt\right)ds \tag{47}$$

$$u(x)=\left(e^{-x}+1\right)+\frac{9}{4}x+2e^{-x}+\frac{1}{8}e^{-2x}-\frac{5}{4}x^2-\frac{1}{6}x^3-\frac{17}{8}+H(x) \tag{48}$$

By replacing u(t) by its expression in the equation (47), we obtain:

$$H(x)=\int_0^x\left(\int_0^s(s-t)\left(\frac{9}{4}t+3e^{-t}+\frac{1}{8}e^{-2t}-\frac{5}{4}t^2-\frac{1}{6}t^3-\frac{9}{8}+H(t)\right)^2dt\right)ds \tag{49}$$

Then

$$\left\{\begin{array}{l} H(x)=\left(-\frac{9}{4}x-2e^{-x}-\frac{1}{8}e^{-2x}+\frac{5}{4}x^2+\frac{1}{6}x^3+\frac{17}{8}\right)+\frac{149\,741}{3072}x+\frac{473}{4}e^{-x}-\frac{117}{128}e^{-2x}- \\ \quad\frac{1}{36}e^{-3x}-\frac{1}{4096}e^{-4x}+\frac{135}{2}xe^{-x}+\frac{3}{32}xe^{-2x}+\frac{33}{2}x^2e^{-x}+\frac{1}{16}x^2e^{-2x}+x^3e^{-x}+\frac{1}{192}x^3e^{-2x}- \\ \quad\frac{3095}{512}x^2+\frac{17}{384}x^3-\frac{27}{128}x^4+\frac{21}{160}x^5-\frac{7}{160}x^6+\frac{13}{3360}x^7+\frac{5}{4032}x^8+\frac{1}{18\,144}x^9-\frac{4324\,439}{36\,864}+ \\ \quad\int_0^x\left(\int_0^s(s-t)\left(\frac{9}{2}t+6e^{-t}+\frac{1}{4}e^{-2t}-\frac{5}{2}t^2-\frac{1}{3}t^3-\frac{9}{4}\right)H(t)dt\right)+\int_0^x\left(\int_0^s(s-t)H^2(t)dt\right) \end{array}\right. \tag{50}$$

Let's make:

$$\left\{\begin{array}{l} H(x)=\sum_{n=0}^{+\infty}H_n(x) \\ H^2(t)=\sum_{n=0}^{+\infty}A_n(t) \end{array}\right. \tag{51}$$

whith $A_n$ were Adomian special polynomial

$$\left\{\begin{array}{l} A_0=H_0^2 \\ A_1=2H_0H_1 \\ A_2=2H_0H_2+H_1^2 \\ A_3=2H_0H_3+2H_1H_2 \end{array}\right. \tag{52}$$

The modified Adomian algorithm for this equation is the following:

$$\begin{cases} H_0(x) = -\frac{9}{4}x - 2e^{-x} - \frac{1}{8}e^{-2x} + \frac{5}{4}x^2 + \frac{1}{6}x^3 + \frac{17}{8} \\ H_1(x) = \frac{149\,741}{3072}x + \frac{473}{4}e^{-x} - \frac{117}{128}e^{-2x} - \frac{1}{36}e^{-3x} - \frac{1}{4096}e^{-4x} + \frac{135}{2}xe^{-x} + \frac{3}{32}xe^{-2x} + \\ \quad \frac{33}{2}x^2e^{-x} + \frac{1}{16}x^2e^{-2x} + x^3e^{-x} + \frac{1}{192}x^3e^{-2x} - \frac{3095}{512}x^2 + \frac{17}{384}x^3 - \frac{27}{128}x^4 + \frac{21}{160}x^5 - \\ \quad \frac{7}{160}x^6 + \frac{13}{3360}x^7 + \frac{5}{4032}x^8 + \frac{1}{18\,144}x^9 - \frac{4324\,439}{36\,864} + \int_0^x \left(\int_0^s (s-t)A_0(t)dt\right) + \\ \quad \int_0^x \left(\int_0^s (s-t)\left(\frac{9}{2}t + 6e^{-t} + \frac{1}{4}e^{-2t} - \frac{5}{2}t^2 - \frac{1}{3}t^3 - \frac{9}{4}\right)H_0(t)dt\right) \\ H_{n+1}(x) = \\ \int_0^x \left(\int_0^s (s-t)\left(\frac{9}{2}t + 6e^{-t} + \frac{1}{4}e^{-2t} - \frac{5}{2}t^2 - \frac{1}{3}t^3 - \frac{9}{4}\right)H_n(t)dt\right) + \int_0^x \left(\int_0^s (s-t)A_n(t)dt\right), \forall n \geq 1 \end{cases} \tag{53}$$

Using $H_0(x)$,calculate of $H_1(x)$ gives:

$$A_0(t) = H_0^2 = \left(-\frac{9}{4}t - 2e^{-t} - \frac{1}{8}e^{-2t} + \frac{5}{4}t^2 + \frac{1}{6}t^3 + \frac{17}{8}\right)^2 \tag{54}$$

and

$$\begin{aligned} H_1(x) = {} & \frac{149\,741}{3072}x + \frac{473}{4}e^{-x} - \frac{117}{128}e^{-2x} - \frac{1}{36}e^{-3x} - \frac{1}{4096}e^{-4x} + \frac{135}{2}xe^{-x} + \frac{3}{32}xe^{-2x} + \\ & \frac{33}{2}x^2e^{-x} + \frac{1}{16}x^2e^{-2x} + x^3e^{-x} + \frac{1}{192}x^3e^{-2x} - \frac{3095}{512}x^2 + \frac{17}{384}x^3 - \frac{27}{128}x^4 + \frac{21}{160}x^5 - \\ & \frac{7}{160}x^6 + \frac{13}{3360}x^7 + \frac{5}{4032}x^8 + \frac{1}{18\,144}x^9 - \frac{4324\,439}{36\,864} + \frac{1}{18\,144}x^9 + \frac{5}{4032}x^8 + \frac{13}{3360}x^7 - \\ & \frac{59}{1440}x^6 + \frac{83}{480}x^5 - \frac{51}{128}x^4 + \frac{1}{192}x^3e^{2(-x)} + \frac{2}{3}x^3e^{-x} + \frac{289}{384}x^3 + \frac{1}{16}x^2e^{2(-x)} + 11x^2e^{-x} - \\ & \frac{8869}{1536}x^2 + \frac{3}{32}xe^{2(-x)} + 45xe^{-x} + \frac{327\,943}{9216}x - \frac{1}{4096}e^{4(-x)} - \frac{1}{54}e^{3(-x)} - \frac{49}{128}e^{2(-x)} + \frac{163}{2}e^{-x} - \\ & \frac{8968\,837}{110\,592} - \frac{1}{9072}x^9 - \frac{5}{2016}x^8 - \frac{13}{1680}x^7 + \frac{61}{720}x^6 - \frac{73}{240}x^5 + \frac{39}{64}x^4 - \frac{1}{96}x^3e^{2(-x)} - \frac{5}{3}x^3e^{-x} - \\ & \frac{51}{64}x^3 - \frac{1}{8}x^2e^{2(-x)} - \frac{55}{2}x^2e^{-x} + \frac{9077}{768}x^2 - \frac{3}{16}xe^{2(-x)} - \frac{225}{2}xe^{-x} - \frac{388\,583}{4608}x + \frac{1}{2048}e^{4(-x)} + \\ & \frac{5}{108}e^{3(-x)} + \frac{83}{64}e^{2(-x)} - \frac{799}{4}e^{-x} + \frac{10\,971\,077}{55\,296} \end{aligned} \tag{55}$$

Thus

$$H_1(x) = \left(-\frac{4324439}{36864} - \frac{8968837}{110592} + \frac{10971077}{55296}\right) + \left(\frac{149741}{3072} + \frac{327943}{9216} - \frac{388583}{4608}\right)x +$$
$$\left(-\frac{3095}{512} - \frac{8869}{1536} + \frac{9077}{768}\right)x^2 + \left(\frac{17}{384} + \frac{289}{384} - \frac{51}{64}\right)x^3 + \left(-\frac{27}{128} - \frac{51}{128} + \frac{39}{64}\right)x^4 +$$
$$\left(\frac{21}{160} + \frac{83}{480} - \frac{73}{240}\right)x^5 + \left(-\frac{7}{160} - \frac{59}{1440} + \frac{61}{720}\right)x^6 + \left(\frac{13}{3360} + \frac{13}{3360} - \frac{13}{1680}\right)x^7 +$$
$$\left(\frac{5}{4032} + \frac{5}{4032} - \frac{5}{2016}\right)x^8 + \left(\frac{1}{18144} + \frac{1}{18144} - \frac{1}{9072}\right)x^9 + \left(\frac{473}{4} + \frac{163}{2} - \frac{799}{4}\right)e^{-x} +$$
$$\left(-\frac{117}{128} - \frac{49}{128} + \frac{83}{64}\right)e^{-2x} + \left(-\frac{1}{36} - \frac{1}{54} + \frac{5}{108}\right)e^{-3x} + \left(-\frac{1}{4096} - \frac{1}{4096} + \frac{1}{2048}\right)e^{-4x} +$$
$$\left(\frac{135}{2} - \frac{225}{2} + 45\right)xe^{-x} + \left(\frac{33}{2} - \frac{55}{2} + 11\right)x^2e^{-x} + \left(\frac{3}{32} + \frac{3}{32} - \frac{3}{16}\right)xe^{-2x} +$$
$$\left(\frac{1}{16} + \frac{1}{16} - \frac{1}{8}\right)x^2e^{-2x} + \left(1 + \frac{2}{3} - \frac{5}{3}\right)x^3e^{-x} + \left(\frac{1}{192} + \frac{1}{192} - \frac{1}{96}\right)x^3e^{-2x}$$

$$H_1(x) = 0 \tag{56}$$

In a recursive way, one deduces

$$\begin{cases} A_n(t) = 0, \forall n \geq 1 \\ \\ H_n(x) = 0, \forall n \geq 1 \end{cases} \tag{57}$$

Then, we have:

$$H(x) = H_0(x) = -\frac{9}{4}x - 2e^{-x} - \frac{1}{8}e^{-2x} + \frac{5}{4}x^2 + \frac{1}{6}x^3 + \frac{17}{8} \tag{58}$$

Thus, the solution of example 3 is:

$$\begin{cases} u(x) = \frac{9}{4}x + 3e^{-x} + \frac{1}{8}e^{-2x} - \frac{5}{4}x^2 - \frac{1}{6}x^3 - \frac{9}{8} + \\ \qquad -\frac{9}{4}x - 2e^{-x} - \frac{1}{8}e^{-2x} + \frac{5}{4}x^2 + \frac{1}{6}x^3 + \frac{17}{8} \\ = e^{-x} + 1 \end{cases} \tag{59}$$

Then, the exact solution:

$$u(x) = 1 + e^{-x} \tag{60}$$

### 4.2.2 Example 4

Consider the following second kind voltera integro-differential equation:

$$\begin{cases} u'(x) = 3x^2 - \frac{x^7}{8} + \int_0^x \frac{t}{x}u^2(t)dt \\ u(0) = 0 \end{cases} \tag{61}$$

**Solving**

By integration equation $(61)$ as $x$,we have:

$$u(x) = x^3 - \frac{1}{64}x^8 + \int_0^x \left( \int_0^s \frac{1}{s} t u^2(t) dt \right) ds \quad (62)$$

Let's make

$$H(x) = \int_0^x \left( \frac{1}{s} \int_0^s t u^2(t) dt \right) ds \quad (63)$$

then the equation $(62)$gives:

$$u(x) = x^3 - \frac{1}{64}x^8 + H(x) \quad (64)$$

Replycing equation $(63)$ inequation $(62)$ gives Adomian canonic form:

$$H(x) = \frac{1}{64}x^8 + \frac{1}{1327\,104}x^{18} - \frac{1}{5408}x^{13} + \int_0^x \left( \frac{1}{s} \int_0^s 2t \left( t^3 - \frac{1}{64}t^8 \right) H(t) dt \right) ds + \int_0^x \left( \frac{1}{s} \int_0^s t H^2(t) dt \right) ds \quad (65)$$

Let's make

$$\begin{cases} H(x) = \sum_{n=0}^{+\infty} H_n(x) \\ H^2(t) = \sum_{n=0}^{+\infty} A_n(t) \end{cases} \quad (66)$$

Equation $(65)$ gives:

$$\sum_{n=0}^{+\infty} H_n(x) = \frac{1}{64}x^8 + \frac{1}{1327\,104}x^{18} - \frac{1}{5408}x^{13} + \sum_{n=0}^{+\infty} \int_0^x \left( \frac{1}{s} \int_0^s 2t \left( t^3 - \frac{1}{64}t^8 \right) H_n(t) dt \right) ds + \sum_{n=0}^{+\infty} \int_0^x \left( \frac{1}{s} \int_0^s t A_n(t) dt \right) ds \quad (67)$$

Then, we have Adomian algorithm modified :

$$\begin{cases} H_0(x) = \frac{1}{64}x^8 \\ H_1(x) = \frac{1}{1327\,104}x^{18} - \frac{1}{5408}x^{13} + \int_0^x \left( \frac{1}{s} \int_0^s 2t \left( t^3 - \frac{1}{64}t^8 \right) H_0(t) dt \right) ds + \int_0^x \left( \frac{1}{s} \int_0^s t A_0(t) dt \right) ds \\ H_{n+1}(x) = \int_0^x \left( \frac{1}{s} \int_0^s 2t \left( t^3 - \frac{1}{64}t^8 \right) H_n(t) dt \right) ds + \int_0^x \left( \frac{1}{s} \int_0^s t A_n(t) dt \right) ds, \forall n \geq 1 \end{cases} \quad (68)$$

where $A_n$ are given by:

$$\begin{cases} A_0 = H_0^2 \\ A_1 = 2H_0H_1 \\ A_2 = 2H_0H_2 + H_1^2 \\ A_3 = 2H_0H_3 + 2H_1H_2 \end{cases} \quad (69)$$

We have

$$H_1(x) = \frac{1}{1327\,104}x^{18} - \frac{1}{5408}x^{13} + \frac{1}{5408}x^{13} - \frac{1}{663\,552}x^{18} + \frac{1}{1327\,104}x^{18} = 0 \quad (70)$$

Then, we have the following expressions

$$\begin{cases} A_n(t) = 0, \forall n \geq 1 \\ \\ H_n(x) = 0, \forall n \geq 1 \end{cases} \tag{71}$$

Then

$$H(x) = H_0(x) = \frac{1}{64}x^8 \tag{72}$$

Hence, the exact solution of example 4 is:

$$u(x) = x^3 - \frac{1}{64}x^8 + \frac{1}{64}x^8 = x^3 \tag{73}$$

### 4.2.3 Example 5

Consider the following second kind Voltera integro-differential equation :

$$\begin{cases} u''(x) = 3e^x - 2e^{2x} + 2\int_0^x e^{x-t}u^2(t)dt \\ u'(0) = 1 \\ u(0) = 1 \end{cases} \tag{74}$$

**Solving**

By two degrees integration of two members of the equation (74) which satisfy initial conditions, we have:

$$u(x) = 3e^x - \frac{1}{2}e^{2x} - x - \frac{3}{2} + 2\int_0^x \left(\int_0^z \left(\int_0^s e^{s-t}u^2(t)dt\right) ds\right) dz \tag{75}$$

Let's make

$$H(x) = 2\int_0^x \left(\int_0^z \left(\int_0^s e^{s-t}u^2(t)dt\right) ds\right) dz \tag{76}$$

we have:

$$u(x) = 3e^x - \frac{1}{2}e^{2x} - x - \frac{3}{2} + H(x) \tag{77}$$

By replacing this last expression in (76), one obtains the canonical form of Adomian following:

$$\begin{cases} H(x) = \frac{17}{4}e^{2x} - \frac{5}{3}e^x - \frac{99}{8}x - \frac{1}{3}e^{3x} + \frac{1}{96}e^{4x} + \frac{1}{2}xe^{2x} - 6x^2e^x + 6xe^x - \frac{29}{4}x^2 - \frac{5}{3}x^3 - \frac{1}{6}x^4 - \frac{217}{96} + \\ \qquad 4\int_0^x \left(\int_0^z \left(\int_0^s e^{s-t}\left(3e^t - \frac{1}{2}e^{2t} - t - \frac{3}{2}\right) H(x)dt\right) ds\right) dz + \\ \qquad 2\int_0^x \left(\int_0^z \left(\int_0^s e^{s-t}\left(H(x)\right)^2 dt\right) ds\right) dz \end{cases} \tag{78}$$

Then, Adomian modified algorithm gives:

$$\begin{cases} H_0(x) = -2e^x + \frac{1}{2}e^{2x} + x + \frac{3}{2} \\ H_1(x) = \frac{1}{3}e^x - \frac{107}{8}x + \frac{15}{4}e^{2x} - \frac{1}{3}e^{3x} + \frac{1}{96}e^{4x} + \frac{1}{2}xe^{2x} - 6x^2e^x + 6xe^x - \frac{29}{4}x^2 - \frac{5}{3}x^3 - \frac{1}{6}x^4 - \\ \frac{361}{96} + 4\int\limits_0^x \left(\int\limits_0^z \left(\int\limits_0^s e^{s-t}\left(3e^t - \frac{1}{2}e^{2t} - t - \frac{3}{2}\right) H_0(t)dt\right) ds\right) dz + 2\int\limits_0^x \left(\int\limits_0^z \left(\int\limits_0^s e^{s-t} A_0(t)dt\right) ds\right) dz \\ H_{n+1}(x) = 4\int\limits_0^x \left(\int\limits_0^z \left(\int\limits_0^s e^{s-t}\left(3e^t - \frac{1}{2}e^{2t} - t - \frac{3}{2}\right) H_n(t)dt\right) ds\right) dz + 2\int\limits_0^x \left(\int\limits_0^z \left(\int\limits_0^s e^{s-t} A_n(t)dt\right) ds\right) dz \end{cases} \tag{79}$$

Using equation (79), we calculate $H_1(x)$ terms:

Thus

$$\begin{aligned} H_1(x) = & \frac{1}{3}e^x - \frac{107}{8}x + \frac{15}{4}e^{2x} - \frac{1}{3}e^{3x} + \frac{1}{96}e^{4x} + \frac{1}{2}xe^{2x} - 6x^2e^x + 6xe^x - \frac{29}{4}x^2 - \frac{5}{3}x^3 - \\ & \frac{1}{6}x^4 - \frac{361}{96} + \frac{1}{3}x^4 + \frac{10}{3}x^3 + 10x^2e^x + \frac{29}{2}x^2 - xe^{2x} - 10xe^x + \frac{337}{12}x - \frac{1}{48}e^{4x} + \\ & \frac{5}{9}e^{3x} - \frac{11}{2}e^{2x} - \frac{23}{3}e^x + \frac{1819}{144} + -\frac{1}{6}x^4 - \frac{5}{3}x^3 - 4x^2e^x - \frac{29}{4}x^2 + \frac{1}{2}xe^{2x} + 4xe^x - \\ & \frac{353}{24}x + \frac{1}{96}e^{4x} - \frac{2}{9}e^{3x} + \frac{7}{4}e^{2x} + \frac{22}{3}e^x - \frac{2555}{288} \\ H_1(x) = & \, 0 \end{aligned} \tag{80}$$

By recursive calculate, we have

$$\begin{cases} A_n(t) = 0, \forall n \geq 1 \\ H_n(x) = 0, \forall n \geq 1 \end{cases} \tag{81}$$

Then

$$H(x) = H_0(x) = -2e^x + \frac{1}{2}e^{2x} + x + \frac{3}{2} \tag{82}$$

Hence, the exact solution of example 5 is:

$$\begin{cases} u(x) = 3e^x - \frac{1}{2}e^{2x} - x - \frac{3}{2} + H(x) \\ u(x) = 3e^x - 2e^x - x + \frac{1}{2}e^{2x} - \frac{1}{2}e^{2x} + x + \frac{3}{2} - \frac{3}{2} \end{cases} \tag{83}$$

Thus

$$u(x) = e^x \tag{84}$$

# 5 Conclusion

In this paper, no specific physical mechanisms are discussed. It is more formal. On the other hand, the aim of this article is therefore not to give a detailed numerical solving of any particular problems of mathematical physics, but rather to mention only the validity of this new approach for solving some linear and nonlinear second kind Voltera integro-differential equations. This approach is based on the use of Adomian decomposition method and method of constants. We have integral party for obtain Adomian canonic form. Then, exact solution is obtain by pursuved classic or modified Adomian method.We have merely tried to give general idea of what kind of problems have been studied. Thus, this method can be used to solve successfuly some integro-differential equations.

## Competing Interests

Authors have declared that no competing interests exist.

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