



An Independent Analysis of the Six Recently Claimed Exomoon Candidates

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Received 2020 July 22; revised 2020 August 6; accepted 2020 August 7; published 2020 September 15

Abstract

It has been recently claimed that KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01, and 3320.01 are exomoon candidates, based on an analysis of their transit timing. Here, we perform an independent investigation, which is framed in terms of three questions: (1) Are there significant transit timing variations (TTVs)? (2) Is there a significant periodic TTV? (3) Is there evidence for a nonzero moon mass? We applied rigorous statistical methods to these questions alongside a reanalysis of the Kepler photometry and find that none of the Kepler objects of interest (KOIs) satisfy our three tests. Specifically, KOIs-268.01 and 3220.01 pass none of the tests and KOIs-303.01, 1888.01, and 1925.01 pass a single test each. Only KOI-2728.01 satisfies two, but fails the cross-validation test for predictions. Further, detailed photodynamical modeling reveals that KOI-2728.01 favors a negative-radius moon (as does KOI-268.01). We also note that we find a significant photoeccentric effect for KOI-1925.01 indicating an eccentric orbit of at least $e > (0.62 \pm 0.06)$. For comparison, we applied the same tests to Kepler-1625b, which reveals that (1) and (3) are passed, but (2) cannot be checked with the cross-validation method used here, due to the limited number of available epochs. In conclusion, we find no compelling evidence for exomoons among the six KOIs. Despite this, we were able to derive exomoon mass upper limits versus semimajor axis, with KOI-3220.01 leading to particularly impressive constraints of $M_S/M_P < 0.4\%$ [2σ] at a similar semimajor to that of the Earth–Moon.

Unified Astronomy Thesaurus concepts: [Exoplanet dynamics \(490\)](#); [Exoplanets \(498\)](#); [Natural satellites \(Extrasolar\) \(483\)](#)

Supporting material: data behind figure

1. Introduction

It has been recently proposed that six Kepler objects of interest (KOIs) host candidate exomoons in Fox & Wiegert (2020). Given the paucity of these objects in the literature, this would represent a major increase in the number of known candidates, as of the time of writing this Letter. For this reason, we provide here an independent analysis of the moon hypothesis for these six: KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01, and 3320.01.

It has been proposed that exomoons could be discovered through a myriad of approaches, such as pulsar timing (Lewis et al. 2008), microlensing (Han & Han 2002), and spectroscopy (Williams & Knacke 2004), but the transit method is somewhat unique in offering the ability to measure the mass and radius of potential moons (see the review by Heller et al. 2014). The mass is available by the study of transit timing effects imparted by the moon upon the planet, which include transit timing variations (TTVs; Sartoretti & Schneider 1999), velocity-induced transit duration variations (TDV-Vs; Kipping 2009a), transit-impact-parameter-induced transit duration variations (TDV-TIPs; Kipping 2009b), and ingress/egress asymmetries (Kipping 2011a). While all of these are generally present, TTVs typically offer the most detectable signal and are the more commonly cataloged timing effect (e.g., see Mazeh et al. 2013).

The case for TTVs is strengthened when one considers that they appear to be common among KOIs (Holzer et al. 2016), potentially indicating a large number of unrevealed exomoons. Indeed, it was recently shown that of 2416 KOIs with a model preference for a periodic TTV, 2198 of them exhibit TTVs and TDVs consistent with an exomoon (Kipping & Teachey 2020). In that paper, among the 2198 aforementioned cases, one finds that KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01, and

3320.01 are all listed in their Table 1 as being fully consistent with an exomoon. Yet despite this, the authors refrained from describing these as exomoon candidates, nor indeed any of the other 2192 cases.

A basic reason for this is that although the TDVs were consistent with an exomoon, no significant detection of them had been made; often only very tentative evidence for TTVs existed. Certainly, in many fields a single type of observational information can be sufficient to securely claim a detection, but the unique challenge facing TTVs is that a considerable number of non-exomoon phenomena can equally cause TTVs. These include, but are not limited to, exotrojans (Ford & Holman 2007), parallax effects (Scharf 2007), eccentricity variations (Kipping 2008), apsidal precession (Jordán & Bakos 2008), starspots (Alonso et al. 2008), stellar proper motion (Rafikov 2009), planetary infall (Hellier et al. 2009), the Applegate effect (Applegate 1992; Watson & Marsh 2010), stellar binarity (Montalto 2010), cadence stroboscopy (Szabó et al. 2013), horseshoe companions (Vokrouhlický & Nesvorný 2014), planet–planet conjunctions (Nesvorný & Vokrouhlický 2014), and near mean motion resonant planets (Agol et al. 2005; Holman & Murray 2005). Planet–planet interactions are particularly common, given the abundance of packed planetary systems found among the Kepler sample (Winn & Fabrycky 2015). On this basis, the existence of TTVs, even periodic TTVs, can be perilous ground upon which to solely base an exomoon claim.

The six candidates claimed by Fox & Wiegert (2020) seem to exhibit TTVs, and thus are indeed consistent with one observational effect of exomoons then. Further, the authors selected targets for exomoons that are plausibly dynamically stable, and honed in on the highest signal-to-noise transits available. That latter point is particularly important since one might expect these KOIs to enable particularly sensitive

searches for exomoons. A search for exomoons is thus interesting around these KOIs in its own right.

Accordingly, in this work, we will interrogate the claim of Fox & Wiegert (2020) for each of the six KOIs argued to be exomoon candidates.

2. Target Data

2.1. Background

In Fox & Wiegert (2020), the authors relied on a catalog of transit timing measurements presented in Holczer et al. (2016). As the Kepler light curves upon which these transit times are derived are publicly available, and the number of objects is fairly small, we elected to derive our own own transit timing estimates.

There are several reasons for doing this. First, the Holczer et al. (2016) transit times are the product of an automated analysis, which made several approximations to expedite their calculation. For example, uncertainties are assigned using an empirical relation rather than actually being formally determined for each object (Holczer et al. 2016). Second, the analysis was conducted prior to the final Kepler Data Release, DR25, and thus does not reflect the most up to date reduction of the Kepler light curves (nor indeed short cadence data where available). Third, the magnitude of the the claim of Fox & Wiegert (2020) warrants a careful independent analysis to interrogate their hypothesis.

2.2. Method Marginalized Light-curve Detrending

To detrend the Kepler light curves, we follow the approach of Teachey & Kipping (2018) and detrend the light curve multiple ways through method marginalized detrending. Of the five detrending approaches used in Teachey & Kipping (2018), we use the same set here except we drop the median filtering approach, as it was found to be the least reliable in that work (see their Figure S7).

We obtained the Simple Aperture Photometry (SAP) and Pre-Data search Conditioning (PDC) DR25 photometric time series from MAST for each KOI. Short cadence (SC) data are used with preference over long cadence (LC), whenever available. We first applied a removal of outliers based on any error flags in the fits file, or 4σ deviations from a 21-point rolling median. Each transit epoch is then detrended independently (where an epoch is centered on the time of transit minimum and spans $\pm 0.5P_p$) with all four algorithms, on both the SAP and PDC time series, giving a total of eight light curves per epoch per target.

We next generated 1000 fake light curves for each method and epoch assuming pure Gaussian noise, in order to test how whitened each light curve is. First, we computed a simple Durbin–Watson statistic (Durbin & Watson 1950) and rejected any light curves that exhibit autocorrelation more than 2σ deviant from white, as characterized by the fake light-curve population. Second, we binned the light curves into progressively larger bins, computed an rms, and then fitted a gradient through the log–log plot of bin size versus rms. This was done for every fake light curve as well as the real, allowing us to again reject any light curves for which the binning properties are more than 2σ deviant from white-noise behavior.

The ≤ 8 surviving light curves (per epoch) were then combined into a so-called method marginalized light curve by calculating an inter-method median at each time stamp, and

propagating the standard deviation between methods into that data point’s error budget through quadrature summation. Last, each epoch is appended to a single file that is used for the subsequent light-curve fits described in Section 3. These are also presented in Figures A1–A3.

3. Analysis

The claim of Fox & Wiegert (2020) is that KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01, and 3320.01 are “exomoon candidates,” which is based upon an analysis of the transit times published by Holczer et al. (2016). Exomoons of transiting planets will also transit their parent star, presenting an additional piece of information that may be used to infer their presence (e.g., Kipping et al. 2013). Nevertheless, we focus on transit timing in what follows since that is the basis upon which the claim of Fox & Wiegert (2020) was made.

To this end, we consider three basic questions for each of the six KOIs under consideration:

- [Q1] Are there statistically significant TTVs?
- [Q2] Is there a statistically significant periodic TTV?
- [Q3] Do the observations support a statistically significant nonzero moon mass?

In the following three subsections, we tackle each of these questions in turn, and then apply the same tests to a previously announced exomoon candidate, Kepler-1625b, in the final part of this section.

3.1. Q1—Are there Significant TTVs?

3.1.1. Inferring the Transit Times

To derive TTVs, we first modeled the transit light curve using the Mandel & Agol (2002) formalism with quadratic limb darkening (using the q_1 – q_2 parameterization of Kipping 2013), and the light-curve integration scheme of Kipping (2010a) to account for LC smearing. Two versions of this model were considered against the data. Model \mathcal{P} assumes a linear ephemeris characterized by an orbital period P and reference time of transit minimum, τ_0 . Model \mathcal{T} allows for TTVs by giving each epoch a unique time of transit minimum, τ_i .

The models are regressed to the method marginalized light curves using a multimodal nested sampling algorithm, MultiNest (Feroz & Hobson 2008; Feroz et al. 2009), providing marginal likelihoods and posterior samples. Priors were set to be uniform for any ephemeris parameters, to within ± 0.5 days of the NASA Exoplanet Archive ephemeris (Akeson et al. 2013). The stellar density (ρ_*) used a log-uniform prior from 10^{-3} to 10^{+3} g cm^{-3} , impact parameter (b) was uniform from 0 to 2, the ratio of radii (p) was uniform from 0 to 1, as were the limb-darkening coefficients q_1 and q_2 . Formally, the orbit is circular, but the ability for the stellar density to vary effectively allows for eccentric orbits since this allows the velocity of the planet to vary. The photometry was modeled with a normal likelihood function, which is justified on the basis that our detrending pre-whitened the data with explicit tests for Gaussianity (see Section 2.2).

Since model \mathcal{T} assigns a unique τ_i to each epoch, this can lead to short-period planets having a large number of total free parameters to explore, which impedes parameter exploration. To circumvent this, we segmented such fits into two subsets of ~ 10 epochs, which was necessary for KOIs-303.01, 1925.01, and 2728.01.

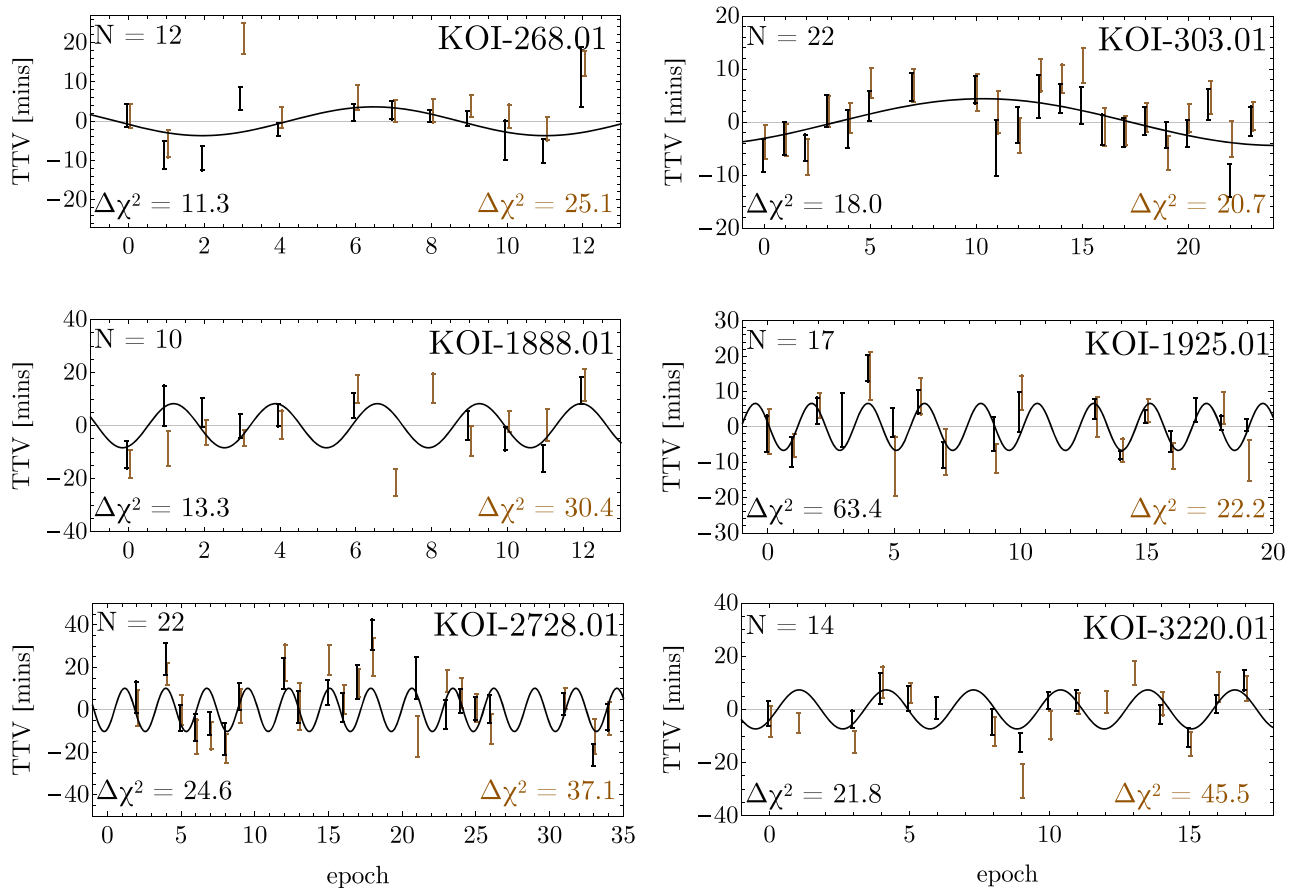


Figure 1. TTVs for the six claimed exomoon candidate hosts of Fox & Wiegert (2020), with our own measurements in black and those of Holczer et al. (2016) in brown. We overplot the best-fitting sinusoid with the χ^2 improvement shown in the lower left corner (and similarly for that of Holczer et al. 2016 in the lower right, although we do not plot the associated sinusoid).

(The data used to create this figure are available.)

3.1.2. Comparison to Times Used by Fox & Wiegert (2020)

From this process, we obtained marginalized posterior distributions for the τ_i parameters for each KOI, which are summarized in the Appendix (Tables A1–A6) and made available at <https://github.com/CoolWorlds/ttvposteriors>. We derived summary statistics for each epoch by computing the median and $\pm 38.1\%$ range. Transit times were converted to TTVs by subtracting the maximum a posteriori ephemeris derived from model \mathcal{P} . For all KOIs, the bulk of the TTVs points exhibited deviations no larger than approximately half an hour, and thus we excluded any points greater than an hour as outliers. These TTVs are shown in Figure 1 and Tables A1–A6, alongside those of Holczer et al. (2016) (and used by Fox & Wiegert 2020) for comparison.

There are noticeable differences between our TTVs and those of Holczer et al. (2016). For every KOI except KOI-1925.01, we find that the $\Delta\chi^2$ improvement of a best sinusoidal fit versus a linear ephemeris is decreased when using our TTVs (see values inset in the panels of Figure 1)—thus largely attenuating the significance of any TTVs.

3.1.3. The Challenge of Defining TTV Significance

Equipped with our new transit times, let us ask whether there are statistically significant variations—a subtle and nontrivial task. One might consider a metric such as the reduced chi-squared, as

utilized by Fox & Wiegert (2020), but since the model is nonlinear then that metric is inappropriate (Andrae et al. 2010).

One might consider comparing the marginal likelihoods evaluated from MultiNest for models \mathcal{T} and \mathcal{P} . However, as noted earlier, model \mathcal{T} overparameterizes the problem here¹ leading to overly conservative estimates for model \mathcal{T} . Indeed, for all six KOIs, model \mathcal{P} would be favored using this approach.

Instead then, one might consider evaluating some statistical measures on the derived TTVs, such as the Bayesian information criterion (BIC; Schwarz 1978). However, those numbers are summary statistics derived from a posterior, and thus applying statistical tests to them is (a) lossy, and (b) demands certain approximate assumptions. It is *lossy* because when one adopts summary statistics of a marginalized distribution, one ignores the full, rich detail of the joint posterior shapes. To avoid such losses, it is preferable to make inferences on the rawest data product that is practical (e.g., see Hogg et al. 2010 for an analogous problem with eccentricities), which in our case would be the photometric light curves. Applying a test like the BIC to summary statistics is also

¹ While this overparameterization is indeed an issue for marginal likelihoods, we point out that it is highly useful for posterior inference since the approach is agnostic as to the cause/shape of possible TTVs and thus lacks any strong model conditionality.

approximate, because it requires an estimate of the maximum likelihood of a hypothesized model, and if that likelihood is derived from summary statistics, then some approximation about the likelihood function describing those summary statistics is necessary (e.g., independent Gaussians).

3.1.4. TTV Significance Tests with the Photometry Directly

A better solution, then, is to apply model comparison tests on the light-curve products, but to avoid using the marginal likelihood due to the parameterization problem of model \mathcal{T} (ultimately that issue is resolved with the photodynamics analysis in the next subsection). Another important limitation is that \mathcal{T} has no predictive power for a held-out epoch, and thus we cannot directly use cross-validation either, at least for Q1.

A basic quantity we can rely on is the maximum likelihood of the light-curve fits, $\hat{\mathcal{L}}$. Since \mathcal{P} is, by definition, a nested model of the more complex model \mathcal{T} , then $\hat{\mathcal{L}}$ will always be greater for \mathcal{T} —the real question is whether the improvement outweighs the expense of the additional complexity that model entails. A common tool for assessing this is the BIC, given by $k \log n - 2 \log \hat{\mathcal{L}}$, where k is the number of parameters estimated by the model and n is the number of data points.² While $\hat{\mathcal{L}}$ and n are well-defined, we again run into an obstacle with k . If we set k as the genuine number of free parameters in the model (i.e., with each epoch requiring an additional parameter), it will again overparameterize the problem. The hypothesis of Fox & Wiegert (2020) is an exomoon whose influence on the TTVs can be fully parameterized by just six additional parameters (Kipping 2011a), assuming a circular orbit moon as expected due to rapid tidal circularization (Porter & Grundy 2011). All six KOIs under consideration here include more than six transit epochs and thus employ more parameters than is necessary to explain an exomoon (or indeed a perturbing planet). On that basis, we would expect a much more optimistic case for TTVs by using $k_{\mathcal{T}} = 6 + k_{\mathcal{P}}$, and indeed a solution well-motivated by our understanding of orbital parameterization. Accordingly, we argue that tests for TTV significance, when a single orbital component is hypothesized, should use $k_{\mathcal{T}} + k_{\mathcal{P}} \rightarrow \min[k_{\mathcal{T}} + k_{\mathcal{P}}, 6 + k_{\mathcal{P}}]$, where the minimum function accounts that in some cases we have less than six epochs.

Proceeding as described, we find that $\text{BIC}_{\mathcal{P}} < \text{BIC}_{\mathcal{T}}$ for KOIs-268.01, 303.01, 1888.01, and 3220.01, indicating no TTVs, whereas KOI-1925.01 and KOI-2728.01 do (see Table 1).

We note a peculiarity about the two positive cases, though. Both are examples of where it was necessary to segment the epochs into two groups, and so we are also able to apply our statistical tests to each segment independently. In doing so, we find that—for both KOIs—one segment shows positive evidence but the other does not. For example, for KOI-1925.01, we obtain $\text{BIC}_{\mathcal{P}-\mathcal{T}} = -36.7$ for the first segment, but $+75.7$ for the second. The first segment includes 7 SC epochs out of 10, whereas the second is 8 out of 9. Thus, this does not appear to offer a good explanation for the large difference. Similarly, for KOI-2728.01 (which has only LC data), the first segment gives $\text{BIC}_{\mathcal{P}-\mathcal{T}} = 30.6$ but the second gives -3.6 . Since exomoons are expected to be strictly periodic signals, it is peculiar for the significance to change versus time, implying a time-dependent amplitude.

² And also note that log is natural, unless stated otherwise.

Table 1
Statistical Tests for Evidence for Exomoons, Using Gravitational Effects Only

KOI	$\text{BIC}_{\mathcal{P}-\mathcal{T}}$ (Q1)	% of Good TTV Predictions (Q2)	$\log K_{M:\lambda}$ (Q3)	(M_S/M_P) at $60 R_P$ [2σ]
268.01	−31.0 [N]	0% [N]	−3.7 [N]	1.7%
303.01	−32.3 [N]	54% [Y]	−1.8 [N]	2.3%
1888.01	−23.7 [N]	17% [N]	+0.6 [Y]	3.3%
1925.01	+93.9 [Y]	41% [N]	−3.0 [N]	5.9%
2728.01	+68.9 [Y]	28% [N]	+0.3 [Y]	4.6%
3220.01	−1.2 [N]	40% [N]	−6.9 [N]	0.39%
K1625b	+3.2	N/A	+1.9	N/A

Note. Answers to the three questions posed at the start of Section 3 are provided in columns 2, 3, and 4, where the first number denotes the statistic used to assess each question, and the Y/N in square brackets is a simple yes/no summary of the posed question. The final column gives a mass ratio upper limit derived. We also show the same tests for Kepler-1625b (Teachey & Kipping 2018), although the cross-validation test is not possible due to the limited number of samples, and a mass upper limit is not provided since this case corresponds to a detection.

3.2. Q2—Is there a Significant Periodic TTV?

Having discussed whether there is statistical evidence for TTVs, we now ask whether there is *periodic* TTV embedded, as expected for exomoons (Sartoretti & Schneider 1999).

We first note that model \mathcal{T} has no predictive capacity for a missing epoch, since every epoch is defined with a unique τ_i independent of the others. As a result, cross-validation—a powerful tool for model selection—is not possible. Although cross-validation cannot be applied to model \mathcal{T} directly, there is a way one can employ it. To do so, we work with the marginalized transit times produced by model \mathcal{T} , rather than the original photometry. This is less preferable for reasons described earlier, but by doing so we can propose a simple sinusoidal model against the derived transit times and use cross-validation to assess its merit. In particular, we propose the following five-parameter model for the transit times:

$$\tau(i) = \tau_0 + iP + A_{\text{TTV}} \sin(\nu_{\text{TTV}}i + \phi_{\text{TTV}}), \quad (1)$$

where the TTV subscript terms control the sinusoidal feature of the model. Note that we applied our model to the transit times, not a list of TTVs. TTVs are defined as deviations from a linear ephemeris, whose parameters are themselves uncertain and indeed degenerate with the sinusoid, especially for slow ν_{TTV} .

Of course, one could use this model on the photometry itself, too (e.g., see Ofir et al. 2018). However, cross-validation generally varies the choice of training and holdout sets, performing many realizations and then inspecting the ensemble for the purposes of model comparison. Since our the photometric fits take around a week to complete on ~ 200 cores, it is not practical to explore this approach in a reasonable time frame.

For our cross-validation, we defined a 20% holdout set from the available epochs. We then took the 80% training set and ran a weighted Lomb–Scargle periodogram (Lomb 1976; Scargle 1982) uniform in frequency. We selected the lowest- χ^2 period and recorded the associated parameters. We also performed a second fit with a simple linear ephemeris as the null model. We then applied both models to the holdout set and asked which one leads to the best prediction in a χ^2 sense.³ We then

³ This implicitly means we approximate the transit time posteriors as being Gaussian.

repeated the entire process, choosing another random group of holdout data, and continued 10^4 times.

The cross-validation results are listed in Table 1. A summary is that that none of the KOIs yield cross-validation results where more than half of the sinusoidal predictions outperform the linear ephemeris model, with the exception of KOI-303.01, which is marginal at 54%. However, KOI-303.01 was found earlier to not statistically favor the existence of TTVs in a more general sense. This is because (a) the cross-validation results are marginal here and give almost even weight to the competing hypotheses, and (b) the earlier test treats the degrees of freedom as being equal to that of an orbiting moon, but here the dimensionality is more restricted.

KOIs-1925.01 and 2728.01 are worth commenting on since those appeared to exhibit significant TTVs (see Section 3.1). As noted in the previous subsection, the case for TTVs seems disparate between the first/second halves of the data set for both objects and indeed the poor cross-validation results make sense in this context. If there are stochastic TTVs (e.g., due to stellar activity), a deterministic model such as a sinusoid or exomoon will indeed fail to make useful predictions, despite the fact large and significant variations exist.

3.3. Q3—Are There Moon-like Timing Variations?

The third and final question requires a model for the dynamical effect of exomoons on the observations. It is not enough for a KOI to exhibit some kind of TTVs, or a periodic TTV signal. This is because exomoons produce more subtle and complex effects into the light curve than the approximate theory of Sartoretti & Schneider (1999), Kipping (2009a), or Kipping (2009b). As explicitly noted in Kipping (2011b), expressions for the TTV (and TDV) waveform caused by an exomoon are approximate and depend upon several approximations. For example, the moon and planet are assumed to experience no acceleration during the transit duration, which requires that $P_S \gg T_{14}$. Given that the KOIs in question have durations up to $\simeq 12$ hr, this implies moons with less than few days orbital period would fail this criteria. Further, exomoons induce other dynamical effects on the light curve besides TTVs—such as TDV-Vs (Kipping 2009a), TDV-TIPs (Kipping 2009b), and ingress/egress asymmetry (Kipping 2011a). While TTVs are a sound place to start an investigation, a detailed consideration of exomoon candidacy should—in our opinion—consider the full details of the hypothesized model.

To address this then, we recommend a photodynamical analysis of the light curve, which allows us to (a) use full photometric time series, rather than lossy derivative products, and (b) fully model the subtle effects exomoons can impart on the light curve.

Photodynamics models the light curve at each time step by evolving an N -body system and calculating the fraction of stellar flux occulted to create a light curve (e.g., see Barros et al. 2015; Almenara et al. 2018; Borkovits et al. 2019). In this work, we use the LUNA algorithm (Kipping 2011a), which is optimized for exomoon fits and extends the Mandel & Agol (2002) formalism.

The claim of Fox & Wiegert (2020) is that these six KOIs exhibit transit timing effects indicative of an exomoon. Transit timing effects are only sensitive to the mass of an exomoon, not its radius; and thus, if the claim of Fox & Wiegert (2020) holds, then there should be some positive evidence for a nonzero exomoon mass. The Fox & Wiegert (2020) claim does not

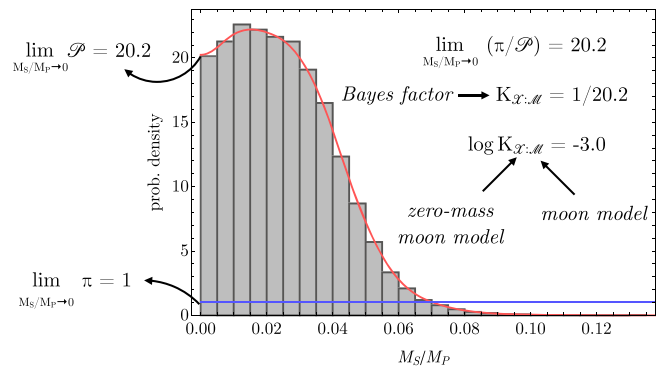


Figure 2. Example of how we calculated the SD ratio of the zero-mass moon model, here for the case of KOI-1925.01. The histogram is calculated from the marginalized posterior distribution, and this is generalized to a continuous function using KDE (red line). We then evaluated the density in the limit of zero mass and compare it to that of the prior. For example, here we find a 20:1 Bayes factor in favor of a zero-mass moon model.

address exomoon radius and so, even though that can be included in our photodynamical model, we leave its inferred value aside for the time being and focus on the photodynamically inferred exomoon mass.

Our moon model included the seven parameters from model \mathcal{P} (P , τ_0 , p , b , ρ_* , q_1 , and q_2) as well as seven additional satellite (“S”) parameters (M_S/M_P , R_S/R_P , a_S/R_P , P_S , ϕ_S , $\cos i_S$, and Ω_S). Note, that only six of these pertain to the gravitational influence on the planet, and were thus counted as penalized terms earlier in Section 3.1, since TTVs are not functionally dependent on R_S/R_P . We adopted uniform priors for all terms except for P_S , which has a log-uniform prior, and consider orbits out to 100 planetary radii. Models were regressed to the light curve using MultiNest, as before.

If there are statistically significant transit timing effects (not just TTVs) that were caused by an exomoon, then the exomoon mass in a photodynamical fit would favor a nonzero value. In our fits, we were careful to not impose any constraint on the exomoon density so that the posteriors can explore masses tending to zero without penalization for unphysical satellite densities. The planetary density, derived using the method of Kipping (2010b), is constrained to be $0.03 \text{ g cm}^{-3} < \rho_P < 150 \text{ g cm}^{-3}$ to prevent the code from exploring unphysical combinations of P_S and a_{SP} .

Mass is a positive definite quantity leading to traditional measures, such as the median, to become positively skewed, and thus posing a challenge to straightforwardly assessing its significance away from zero. To resolve this, one might first consider using something like a Lucy & Sweeney (1971) test, but a more rigorous Bayesian approach is described in Jontof-Hutter et al. (2015) via the Savage–Dickey (SD) ratio Dickey (1971), and we follow that approach here. We evaluated the SD ratio by comparing the posterior density at $M_S/M_P = 0$ versus the prior (uniform) with an example illustrated in Figure 2.

The SD ratio allows for an estimate of the Bayes factor for nested models. Here, then, we compare the original full moon model, dubbed model \mathcal{M} , against the same model but with no mass effects (dubbed \mathcal{X}). Since the Fox & Wiegert (2020) claim concerns transit timing effects due to an exomoon, implying a nonzero exomoon mass, then this act directly evaluates the case for their claim in a Bayesian framework with a self-consistent, photodynamical model.

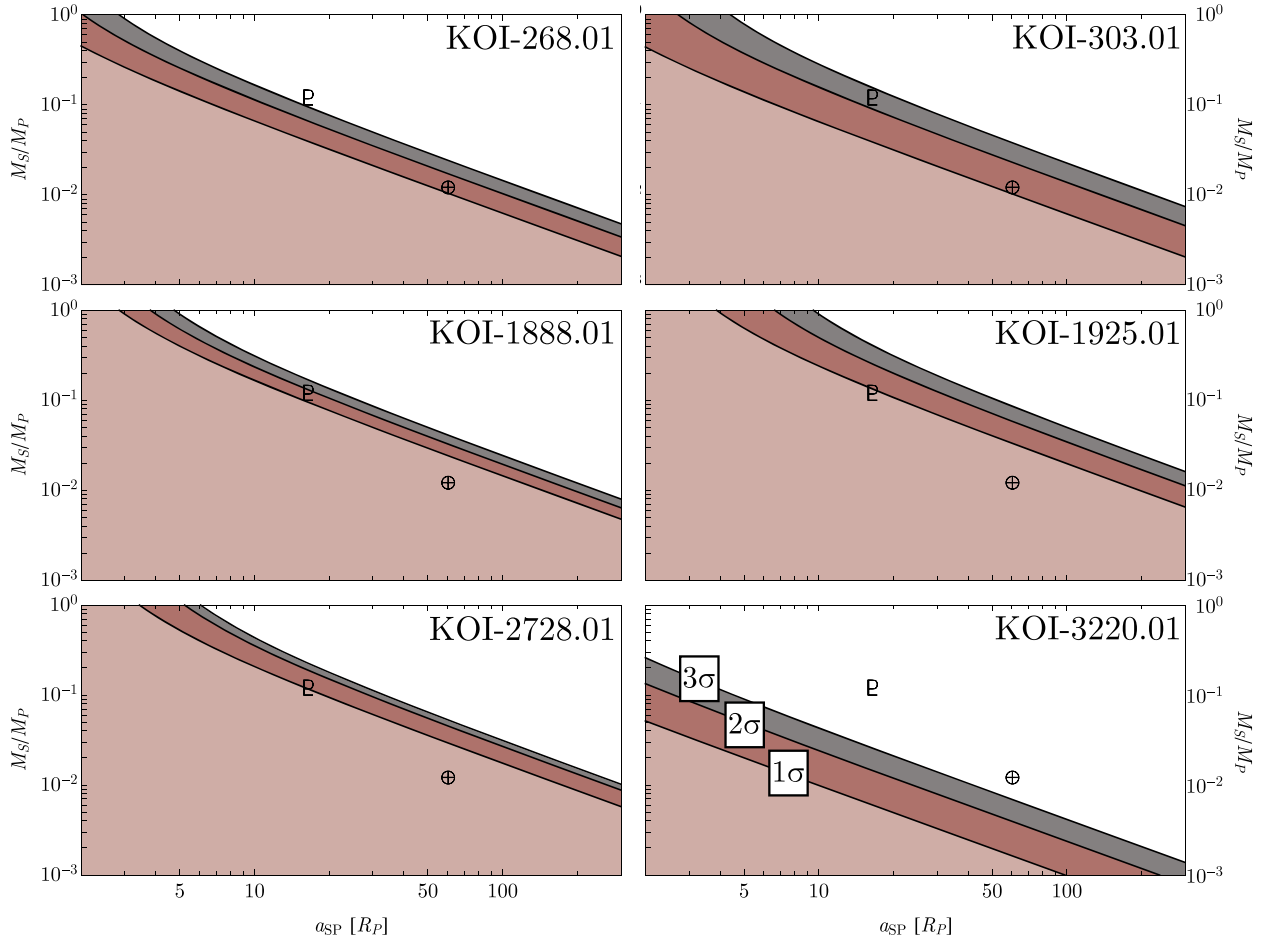


Figure 3. Mass limits to exomoons for KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01 and 3320.01. Although we find no evidence for exomoon candidates, the high signal to noise of these transits allows for strong upper limits. We denote the position of Pluto–Charon and the Earth–Moon on the diagram for context.

Table 1 shows the results of this exercise, where we find a preference for zero-mass moon models for KOIs-268.01, 303.01, 1925.01, and 3220.01 and very marginal preferences for a positive mass for KOI-1888.01 and KOI-2728.01.

3.4. Other Insights from the Photodynamical Fits

Some other notable aspects of the results are briefly discussed. For KOIs 268.01, 303.01, 2728.01, and 3220.01, the agreement between the light-curve stellar density and that from an isochrone analysis⁴ are within 2σ . For KOI-1888.01, it is a little worse at 3σ . But for KOI-1925.01 the difference is pronounced, with the log of the ratio between them found to be $\log(\rho_{*,LC}/\rho_{*,isochrones}) = 2.2 \pm 0.3$, implying a minimum orbital eccentricity via the photoeccentric effect (Dawson & Johnson 2012) of 0.62 ± 0.06 —which would pose a significant challenge for an exomoon due the truncation of the Hill sphere at periape (Domingos et al. 2006). We also verified this by taking the results from model \mathcal{T} , evaluating a kernel density estimation (KDE) of each segment’s density ratio posterior, taking the product of the two, numerically normalizing, and then evaluating the median and standard deviation to give $\log(\rho_{*,LC}/\rho_{*,isochrones}) = 2.3 \pm 0.2$. On this basis, we assert with confidence that the densities are in tension for

KOI-1925.01 and the object likely maintains an eccentricity in excess of 0.6.

We also note that our exomoon fits permit negative-radius moons, which translate to inverted transits, and indeed some of our fits converge to such unphysical solutions. In particular, KOI-268.01 and 2728.01 both strongly favor negative-radius moons.

3.5. Application to Kepler-1625b

For completion, we decided to apply these tests to data used to claim an exomoon candidate by Teachey & Kipping (2018). The results are shown in Table 1. Kepler-1625b passes Q1 and Q3, but we are able to evaluate Q2. The reason for this is that with just four epochs, the act of regressing the five-parameter model given by Equation (1) to the data leads to an overdetermined system. This is exacerbated if we drop an epoch for cross-validation purposes. Nevertheless, we find that the results of the tests described here pose no challenge to the candidacy of Kepler-1625b i.

4. Discussion

In this work, we have conducted an independent examination of the claim of Fox & Wiegert (2020) that KOIs-268.01, 303.01, 1888.01, 1925.01, 2728.01, and 3320.01 are “exomoon candidates.” As the claim is based on transit timing effects

⁴ This is achieved by using the Gaia DR2 parallax, Kepler magnitude, and Mathur et al. (2017) DR25 stellar atmospheric properties of each KOI into isochrones (Morton 2015).

only, we have primarily framed our investigation in those same terms.

We structure our investigation in terms of three basic questions: (1) Are there significant TTVs? (2) Is there a significant periodic TTV? (3) Is there a statistically significant nonzero exomoon mass? It’s worth noting that the third criterion is a standard test used by the “Hunt for Exomoons with Kepler” project, namely, criterion B2a (Kipping et al. 2013, 2015). Rather than rely on the catalog transit times of Holczer et al. (2016), we elected to infer our own times using method marginalized detrending of the latest Kepler data products and incorporating SC time series where available.

The results of these three questions/tests are summarized in Table 1. We find that KOIs-268.01 and 3220.01 result in a “no” for all three questions. KOIs-303.01, 1888.01, and 1925.01 pass a single test each, although a different one in each case. The analysis of this work thus concludes that these five KOIs are not exomoon candidates.

Only KOI-2728.01 passes two of the three, failing the cross-validation test when we ask if the periodic TTV has predictive capability. Specifically, when we split the transit times into an 80:20 training:holdout set, we find that the hypothesis of a periodic sinusoid defeats the predictions of the null hypothesis (a linear ephemeris) in only 28% of the draws. One explanation

would be that the TTVs are significant but are stochastic, perhaps caused by stellar activity (Alonso et al. 2008), thus failing the periodic prediction test. As an additional point of concern, KOI-2728.01 favors a negative-radius moon when fit with a photodynamical exomoon model. On this basis, we do not consider there to be a good case for KOI-2728.01 being an exomoon candidate. Upper limits are presented for each in Figure 3.

It is important that we continue to search for exomoons, but they are unquestionably very challenging objects to detect; not only at the hairy edge of Kepler’s sensitivity, but also plagued by a myriad of false positives when considering a single observable quantity, such as TTV. On the other hand their existence is equally unquestionable, planets surely do have moons (!), but we caution that they demand very high levels of care and statistical rigor.

D.M.K. is supported by the Alfred P. Sloan Foundation. Thanks to the anonymous reviewer for a constructive report. Special thanks to Tom Widdowson, Mark Sloan, Laura Sanborn, Douglas Daughaday, Andrew Jones, Jason Allen, Marc Lijoi, Elena West, and Tristan Zajonc.

Appendix

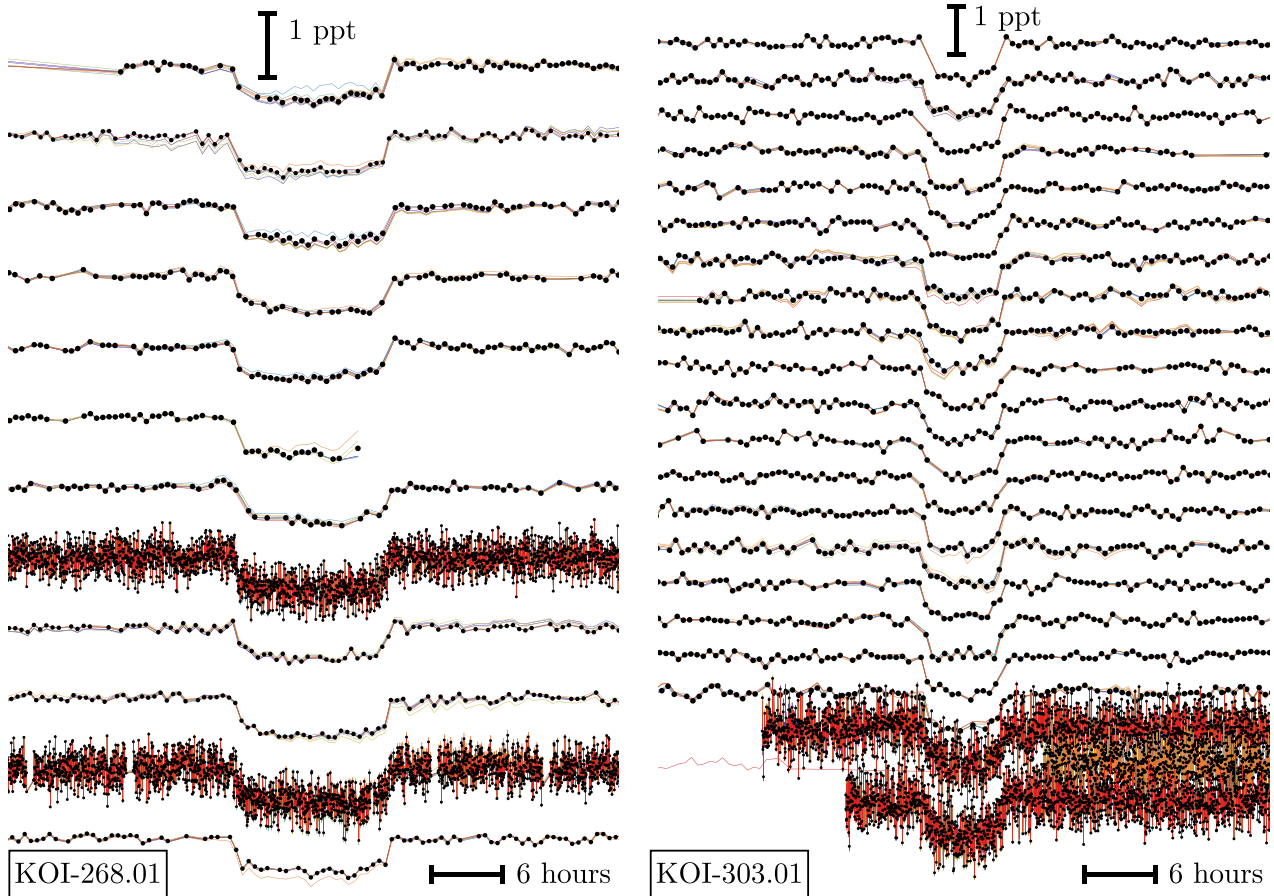


Figure A1. Method marginalized detrended light curves for KOI-268.01 and KOI-303.01. The eight colored continuous lines show the eight different independent detrendings of each epoch, which are then combined together to form the method marginalized time series (black points).

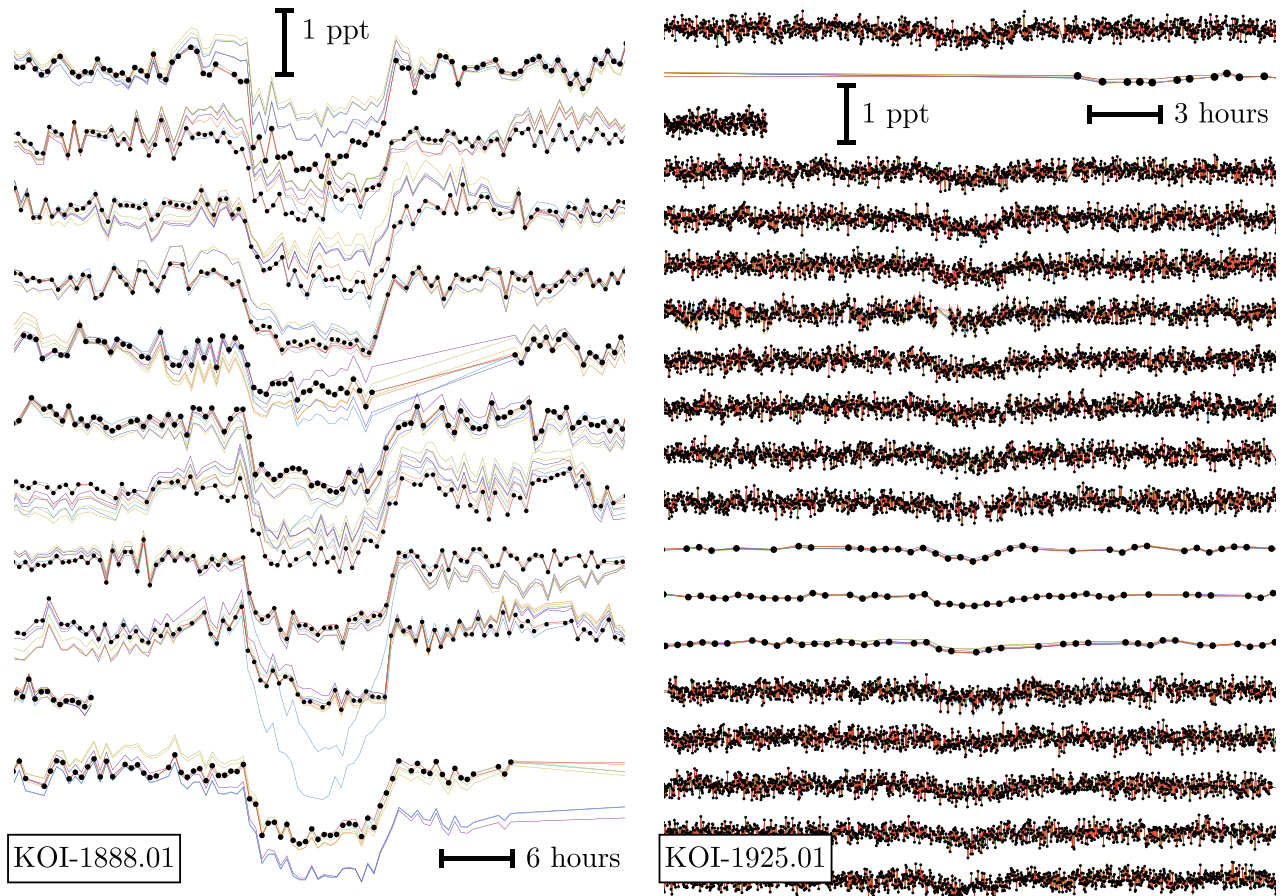


Figure A2. Method marginalized detrended light curves for KOI-1888.01 and KOI-1925.01. The eight colored continuous lines show the eight different independent detrendings of each epoch, which are then combined together to form the method marginalized time series (black points).

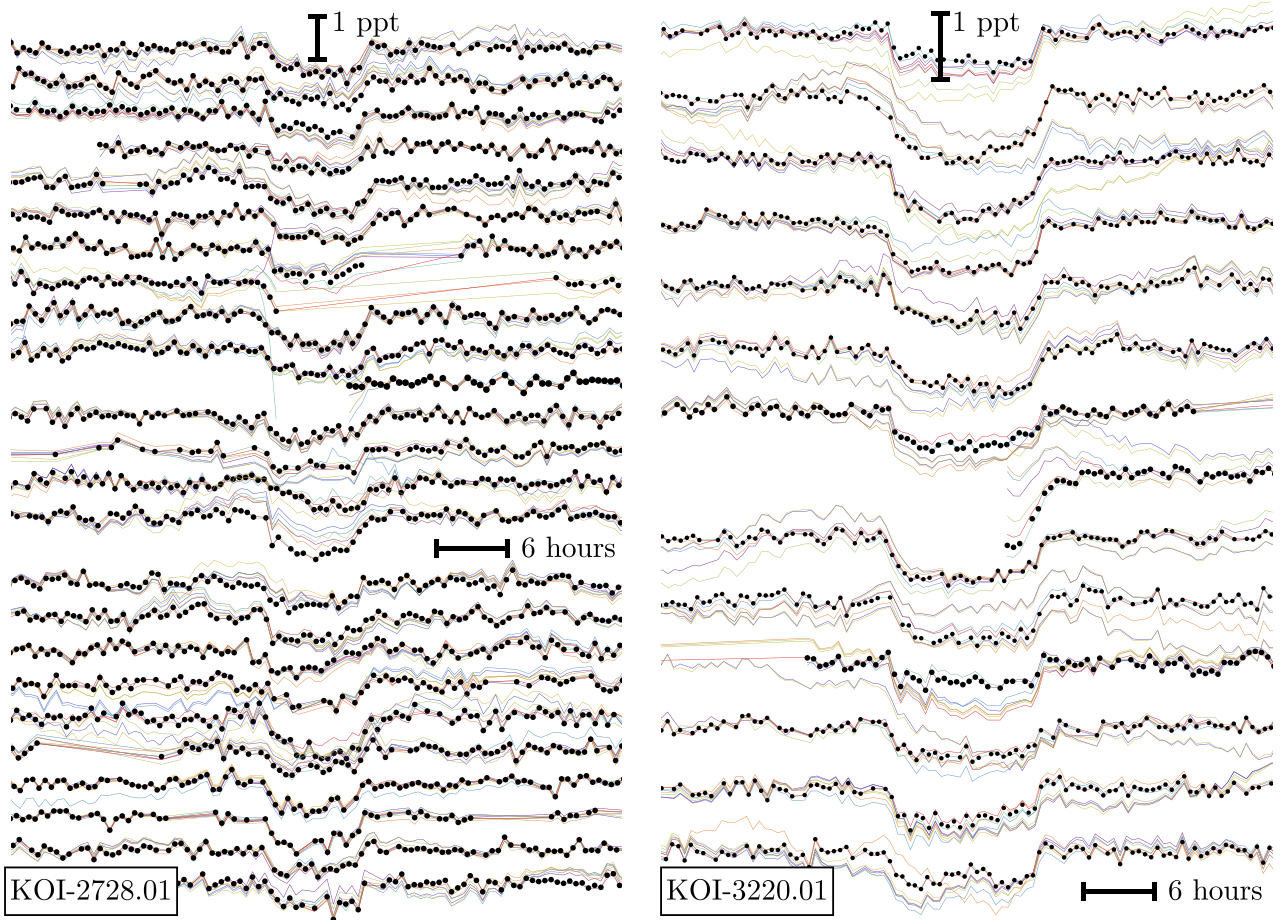


Figure A3. Method marginalized detrended light curves for KOI-2728.01 and KOI-3220.01. The eight colored continuous lines show the eight different independent detrendings of each epoch, which are then combined together to form the method marginalized time series (black points).

Table A1
Transit Timing of KOI-268.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV _{<i>i</i>} (minutes)
0	$8.9346^{+0.0020}_{-0.0020}$	$1.5^{+2.9}_{-2.9}$
1	$119.3058^{+0.0026}_{-0.0023}$	$-8.6^{+3.8}_{-3.3}$
2	$229.6835^{+0.0023}_{-0.0021}$	$-9.4^{+3.3}_{-3.0}$
3	$340.0722^{+0.0021}_{-0.0020}$	$5.8^{+3.0}_{-2.9}$
4	$450.4449^{+0.0011}_{-0.0011}$	$-2.1^{+1.6}_{-1.6}$
6	$671.2044^{+0.0014}_{-0.0015}$	$2.2^{+2.0}_{-2.1}$
7	$781.5830^{+0.0016}_{-0.0016}$	$2.8^{+2.4}_{-2.3}$
8	$891.9602^{+0.0010}_{-0.0011}$	$1.4^{+1.5}_{-1.5}$
9	$1002.3379^{+0.0013}_{-0.0014}$	$0.7^{+1.9}_{-2.0}$
10	$1112.7123^{+0.0036}_{-0.0034}$	$-4.8^{+5.1}_{-4.9}$
11	$1223.0886^{+0.0021}_{-0.0022}$	$-7.5^{+3.0}_{-3.2}$
12	$1333.4798^{+0.0050}_{-0.0056}$	$11.0^{+7.2}_{-8.1}$

Note. Transit times are quoted as $\text{BJD}_{\text{UTC}} - 2,455,000$. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself).

Table A2
Transit Timing of KOI-303.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV _{<i>i</i>} (minutes)
0	6.3664 ^{+0.0022} _{-0.0021}	-6.2 ^{+3.2} _{-3.1}
1	67.2969 ^{+0.0021} _{-0.0022}	-3.1 ^{+3.1} _{-3.2}
2	128.2240 ^{+0.0017} _{-0.0018}	-4.9 ^{+2.4} _{-2.6}
3	189.1571 ^{+0.0021} _{-0.0020}	2.1 ^{+3.1} _{-2.9}
4	250.0830 ^{+0.0025} _{-0.0025}	-1.4 ^{+3.5} _{-3.6}
5	311.0143 ^{+0.0019} _{-0.0020}	3.0 ^{+2.8} _{-2.8}
7	432.8734 ^{+0.0018} _{-0.0018}	6.7 ^{+2.7} _{-2.6}
10	615.6579 ^{+0.0018} _{-0.0018}	6.1 ^{+2.6} _{-2.6}
11	676.5786 ^{+0.0040} _{-0.0034}	-4.9 ^{+5.7} _{-4.9}
12	737.5099 ^{+0.0023} _{-0.0024}	-0.6 ^{+3.3} _{-3.4}
13	798.4419 ^{+0.0027} _{-0.0029}	4.8 ^{+3.8} _{-4.2}
14	859.3700 ^{+0.0019} _{-0.0019}	4.5 ^{+2.8} _{-2.7}
15	920.2974 ^{+0.0026} _{-0.0023}	3.2 ^{+3.7} _{-3.3}
16	981.2224 ^{+0.0019} _{-0.0021}	-1.4 ^{+2.7} _{-3.0}
17	1042.1503 ^{+0.0019} _{-0.0019}	-2.0 ^{+2.7} _{-2.7}
18	1103.0801 ^{+0.0019} _{-0.0018}	0.1 ^{+2.7} _{-2.6}
19	1164.0066 ^{+0.0017} _{-0.0017}	-2.4 ^{+2.4} _{-2.4}
20	1224.9351 ^{+0.0018} _{-0.0018}	-2.1 ^{+2.6} _{-2.6}
21	1285.8672 ^{+0.0019} _{-0.0021}	3.3 ^{+2.7} _{-3.1}
22	1346.7855 ^{+0.0023} _{-0.0020}	-11.0 ^{+3.3} _{-2.9}
23	1407.7215 ^{+0.0019} _{-0.0019}	0.1 ^{+2.8} _{-2.8}

Note. Transit times are quoted as BJD_{UTC} - 2, 455, 000. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself). Horizontal line denotes the split between the two segments used.

Table A3
Transit Timing of KOI-1888.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV _{<i>i</i>} (minutes)
0	-32.8210 ^{+0.0035} _{-0.0037}	-11.0 ^{+5.0} _{-5.5}
1	87.2100 ^{+0.0055} _{-0.0051}	7.5 ^{+7.9} _{-7.4}
2	207.2265 ^{+0.0041} _{-0.0036}	4.9 ^{+5.9} _{-5.2}
3	327.2412 ^{+0.0032} _{-0.0030}	-0.2 ^{+4.6} _{-4.3}
4	447.2623 ^{+0.0029} _{-0.0029}	4.0 ^{+4.2} _{-4.2}
6	687.3014 ^{+0.0033} _{-0.0032}	7.8 ^{+4.8} _{-4.6}
9	1047.3508 ^{+0.0038} _{-0.0037}	0.1 ^{+5.5} _{-5.4}
10	1167.3657 ^{+0.0031} _{-0.0030}	-4.8 ^{+4.5} _{-4.4}
11	1287.3787 ^{+0.0035} _{-0.0035}	-12.0 ^{+5.0} _{-5.1}
12	1407.4147 ^{+0.0035} _{-0.0035}	13.0 ^{+5.1} _{-5.1}

Note. Transit times are quoted as BJD_{UTC} - 2, 455, 000. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself).

Table A4
Transit Timing of KOI-1925.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV _{<i>i</i>} (minutes)
0	12.0806 ^{+0.0026} _{-0.0045}	-2.0 ^{+3.8} _{-6.5}
1	81.0356 ^{+0.0030} _{-0.0029}	-7.0 ^{+4.3} _{-4.2}
2	150.0020 ^{+0.0026} _{-0.0027}	4.5 ^{+3.7} _{-3.8}
3	218.9587 ^{+0.0054} _{-0.0052}	2.0 ^{+7.8} _{-7.6}
4	287.9274 ^{+0.0026} _{-0.0026}	17.0 ^{+3.8} _{-3.7}
5	356.8752 ^{+0.0031} _{-0.0026}	1.3 ^{+4.5} _{-3.3}
6	425.8377 ^{+0.0025} _{-0.0022}	7.2 ^{+3.5} _{-3.1}
7	494.7856 ^{+0.0023} _{-0.0028}	-8.1 ^{+3.3} _{-4.1}
9	632.7067 ^{+0.0025} _{-0.0041}	-2.0 ^{+3.6} _{-5.9}
10	701.6695 ^{+0.0037} _{-0.0042}	4.2 ^{+5.3} _{-6.1}
13	908.5455 ^{+0.0015} _{-0.0023}	5.1 ^{+2.2} _{-3.3}
14	977.49487 ^{+0.00095} _{-0.00077}	-8.0 ^{+1.4} _{-1.1}
15	1046.4610 ^{+0.0012} _{-0.0012}	3.0 ^{+1.8} _{-1.8}
16	1115.4145 ^{+0.0013} _{-0.0027}	-4.1 ^{+1.9} _{-3.8}
17	1184.3791 ^{+0.0015} _{-0.0033}	4.8 ^{+2.1} _{-4.7}
18	1253.3350 ^{+0.0014} _{-0.0015}	1.2 ^{+2.0} _{-2.1}
19	1322.2931 ^{+0.0013} _{-0.0010}	0.5 ^{+1.9} _{-1.5}

Note. Transit times are quoted as BJD_{UTC}—2,455,000. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself). Horizontal line denotes the split between the two segments used.

Table A5
Transit Timing of KOI-2728.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV _{<i>i</i>} (minutes)
2	49.3312 ^{+0.0055} _{-0.0048}	5.0 ^{+7.9} _{-6.9}
4	134.0463 ^{+0.0055} _{-0.0050}	23.0 ^{+7.9} _{-7.2}
5	176.3782 ^{+0.0043} _{-0.0040}	-4.5 ^{+6.2} _{-5.7}
6	218.7264 ^{+0.0042} _{-0.0047}	-8.8 ^{+6.1} _{-6.7}
7	261.0789 ^{+0.0037} _{-0.0037}	-6.9 ^{+5.3} _{-5.3}
8	303.4251 ^{+0.0054} _{-0.0051}	-14.0 ^{+7.8} _{-7.3}
9	345.7901 ^{+0.0045} _{-0.0043}	5.9 ^{+6.4} _{-6.2}
12	472.8512 ^{+0.0051} _{-0.0049}	17.0 ^{+7.4} _{-7.1}
13	515.1917 ^{+0.0061} _{-0.0045}	1.2 ^{+8.7} _{-6.4}
15	599.8986 ^{+0.0041} _{-0.0041}	8.0 ^{+5.9} _{-5.9}
16	642.2450 ^{+0.0047} _{-0.0046}	1.0 ^{+6.8} _{-6.6}
17	684.6047 ^{+0.0056} _{-0.0055}	13.0 ^{+8.1} _{-8.0}
18	726.9712 ^{+0.0047} _{-0.0053}	35.0 ^{+6.7} _{-7.6}
21	854.0108 ^{+0.0068} _{-0.0071}	15.0 ^{+9.7} _{-10.0}
23	938.7014 ^{+0.0048} _{-0.0047}	-1.6 ^{+6.9} _{-6.8}
24	981.0569 ^{+0.0038} _{-0.0039}	4.5 ^{+5.5} _{-5.6}
25	1023.4057 ^{+0.0039} _{-0.0038}	1.1 ^{+5.6} _{-5.5}
26	1065.7568 ^{+0.0041} _{-0.0048}	1.0 ^{+6.0} _{-6.9}
31	1277.5145 ^{+0.0034} _{-0.0037}	3.7 ^{+4.9} _{-5.3}
33	1362.2002 ^{+0.0034} _{-0.0038}	-20.0 ^{+4.9} _{-5.5}
34	1404.5638 ^{+0.0042} _{-0.0044}	-2.5 ^{+6.1} _{-6.3}

Note. Transit times are quoted as BJD_{UTC}—2,455,000. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself). Horizontal line denotes the split between the two segments used.

Table A6Transit Timing of KOI-3220.01 Derived in This Work Using Model \mathcal{T}

Epoch	τ_i	TTV $_i$ (minutes)
0	$-6.1363^{+0.0033}_{-0.0032}$	$-1.7^{+4.7}_{-4.6}$
3	$238.1103^{+0.0022}_{-0.0024}$	$-3.7^{+3.2}_{-3.4}$
4	$319.5344^{+0.0048}_{-0.0031}$	$7.9^{+6.9}_{-4.5}$
5	$400.9478^{+0.0032}_{-0.0031}$	$4.1^{+4.6}_{-4.5}$
6	$482.3613^{+0.0028}_{-0.0029}$	$0.5^{+4.1}_{-4.2}$
8	$645.1897^{+0.0032}_{-0.0036}$	$-4.7^{+4.7}_{-5.1}$
9	$726.6003^{+0.0024}_{-0.0025}$	$-13.0^{+3.5}_{-3.6}$
10	$808.0274^{+0.0022}_{-0.0022}$	$3.4^{+3.1}_{-3.2}$
11	$889.4434^{+0.0027}_{-0.0027}$	$3.3^{+4.0}_{-3.8}$
14	$1133.6878^{+0.0025}_{-0.0024}$	$-1.9^{+3.6}_{-3.4}$
15	$1215.0977^{+0.0025}_{-0.0024}$	$-11.0^{+3.6}_{-3.5}$
16	$1296.5225^{+0.0023}_{-0.0023}$	$2.1^{+3.5}_{-3.3}$
17	$1377.9448^{+0.0028}_{-0.0026}$	$11.0^{+4.0}_{-3.7}$

Note. Transit times are quoted as BJD_{UTC}−2,455,000. TTVs are defined against the maximum a posteriori ephemeris obtained from model \mathcal{P} . Central values are the median of the marginalized posterior distribution and the uncertainties represent the $\pm 34.1\%$ range (TTVs do not propagate the uncertainty of the ephemeris itself).

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- Agol, E., Steffen, J., Sari, R., & Clarkson, W. 2005, *MNRAS*, 359, 567
- Akeson, R. L., Chen, X., Ciardi, D., et al. 2013, *PASP*, 125, 989
- Almenara, J. M., Díaz, R. F., Dorn, C., Bonfils, X., & Udry, S. 2018, *MNRAS*, 478, 460
- Alonso, R., Auvergne, M., Baglin, A., et al. 2008, *A&A*, 482, L21
- Andrae, R., Schulze-Hartung, T., & Melchior, P. 2010, arXiv:1012.3754
- Applegate, J. H. 1992, *ApJ*, 385, 621
- Barros, S. C. C., Almenara, J. M., Demangeon, O., et al. 2015, *MNRAS*, 454, 4267
- Borkovits, T., Rappaport, S., Kaye, T., et al. 2019, *MNRAS*, 483, 1934
- Dawson, R., & Johnson, J. A. 2012, *ApJ*, 756, 122
- Dickey, J. M. 1971, *Ann. Math. Stat.*, 42, 204
- Domingos, R. C., Winter, O. C., & Yokoyama, T. 2006, *MNRAS*, 373, 1227
- Durbin, J., & Watson, G. S. 1950, *Biometrika*, 37, 409
- Feroz, F., & Hobson, M. P. 2008, *MNRAS*, 384, 449
- Feroz, F., Hobson, M. P., & Bridges, M. 2009, *MNRAS*, 398, 1601
- Ford, E. B., & Holman, M. J. 2007, *ApJL*, 664, L51
- Fox, C., & Wiegert, P. 2020, *MNRAS*, submitted (arXiv:2006.12977)
- Han, C., & Han, W. 2002, *ApJ*, 580, 490
- Heller, R., Williams, D., Kipping, D., et al. 2014, *AsBio*, 14, 798
- Hellier, C., Anderson, D. R., Cameron, A. C., et al. 2009, *Natur*, 460, 1098
- Hogg, D. W., Myers, A. D., & Bovy, J. 2010, *ApJ*, 725, 2166
- Holczer, T., Mazeh, T., Nachmani, G., et al. 2016, *ApJS*, 225, 9
- Holman, M. J., & Murray, N. W. 2005, *Sci*, 307, 1288
- Jontof-Hutter, D., Rowe, J. F., Lissauer, J. J., Fabrycky, D. C., & Ford, E. B. 2015, *Natur*, 522, 321
- Jordán, A., & Bakos, G. Á. 2008, *ApJ*, 685, 543
- Kipping, D. M. 2008, *MNRAS*, 389, 1383
- Kipping, D. M. 2009a, *MNRAS*, 392, 181
- Kipping, D. M. 2009b, *MNRAS*, 396, 1797
- Kipping, D. M. 2010a, *MNRAS*, 408, 1785
- Kipping, D. M. 2010b, *MNRAS*, 409, L119
- Kipping, D. M. 2011a, *MNRAS*, 416, 689
- Kipping, D. M. 2011b, PhD thesis, Univ. College London
- Kipping, D. M. 2013, *MNRAS*, 235, 2152
- Kipping, D. M., Hartman, J., Buchhave, L. A., et al. 2013, *ApJ*, 770, 101
- Kipping, D. M., Schmitt, A. R., Huang, X., et al. 2015, *ApJ*, 813, 14
- Kipping, D. M., & Teachey, A. 2020, *MNRAS*, submitted (arXiv:2004.04230)
- Lewis, K. M., Sackett, P. D., & Mardling, R. A. 2008, *ApJL*, 685, L153
- Lomb, N. R. 1976, *Ap&SS*, 39, 447
- Lucy, L. B., & Sweeney, M. A. 1971, *AJ*, 76, 544
- Mandel, K., & Agol, A. 2002, *ApJ*, 580, 171
- Mathur, S., Huber, D., Batalha, N. M., et al. 2017, *ApJS*, 229, 30
- Mazeh, T., Nachmani, G., Holczer, T., et al. 2013, *ApJS*, 208, 16
- Montalto, M. 2010, *A&A*, 521, A60
- Morton, T. D. 2015, isochrones: Stellar model grid package, Astrophysics Source Code Library, ascl:1503.010
- Nesvorný, D., & Vokrouhlický, D. 2014, *ApJ*, 790, 58
- Ofir, A., Xie, J.-W., Jiang, C.-F., Sari, R., & Aharonson, O. 2018, *ApJS*, 234, 9
- Porter, S. B., & Grundy, W. B. 2011, *ApJL*, 736, L14
- Rafikov, R. R. 2009, *ApJ*, 700, 965
- Sartoretti, P., & Schneider, J. 1999, *A&AS*, 134, 553
- Scargle, J. D. 1982, *ApJ*, 263, 835
- Scharf, C. A. 2007, *ApJ*, 661, 1218
- Schwarz, G. E. 1978, *AnSta*, 6, 461
- Szabó, R., Szabó, Gy. M., Dálya, G., et al. 2013, *A&A*, 533, A17
- Teachey, A., & Kipping, D. M. 2018, *SciA*, 4, eaav1784
- Vokrouhlický, D., & Nesvorný, D. 2014, *ApJ*, 791, 6
- Watson, C. A., & Marsh, T. R. 2010, *MNRAS*, 405, 2037
- Williams, D. M., & Knacke, R. F. 2004, *AsBio*, 4, 400
- Winn, J. N., & Fabrycky, D. C. 2015, *ARA&A*, 53, 409