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Differential Equations Applied to Atomic Force Microscopy: A Strategy to Classical Mechanics Teaching in Undergraduation

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Authors' contributions

This work was carried out in collaboration between all authors. Authors GACL and HDFF designed the study, wrote the protocol and supervised the work and wrote the first draft of the manuscript. Author EAC managed the analyses of the study. Author GACL managed the literature searches and edited the manuscript. All authors read and approved the final manuscript.

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ABSTRACT

Aims: To purpose a didactic mechanism for the teaching of the physical and mathematical concepts with respect to the mechanical oscillations, which is commonly addressed in undergraduate courses in Physics, in the chair of Classical Mechanics or correlated ones.
Place and Duration of Study: The study was performed in the Physics Department of Federal

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University of Amapá, Macapá-AP, Brazil during the year of 2015.
Study Design: In order to give an application of the content “mechanical oscillations” we first introduce the theory of Atomic Force Microscope. Then we get the equation of motion for the AFM cantilever, which models a non-linear driven harmonic oscillator with damping, and solve it considering different physical situations.
Methodology: We emphasize the methods of resolution of differential equations, attempting to relate its applications in condensed matter physics, in particular to atomic force microscopy (AFM).
Results: Throughout the motion equation for the AFM cantilever, we studied the solution of the equations for the cases of mechanical equilibrium, simple oscillations, damped oscillations, forced oscillations and damped and forced oscillations.
Conclusions: By applying the differential equations to a recent and instigating research area, the subject becomes better understood by students, gaining great results of learning. This purpose, however, need further work to confirm its educational potential by applying to an undergraduation class.

Keywords: Mechanical oscillations; atomic force microscopy; solution methods of ODE’s; physics teaching.

1. INTRODUCTION

One of the first contact of undergraduate physics students with the application of differential equations take place in the I-IV Basic Physics disciplines, more specifically in Basic Physics II, when is studied the oscillatory motion. In Brazil, as well as in most world universities, the set of these four courses is known as “basic cycle”, but they are taught globally in all Physics Courses. However, in a more profound way, this contact takes place in the discipline of Classical Mechanics, as part of the study of mechanical oscillations, usually addressed as the second topic of the course, following to the introduction of Newton’s laws [1].

This chapter of undergraduation plays a crucial role in the curricular performance of the student, once that Classical Mechanics discipline is often the first, in Physics courses, which follows from the “basic cycle”, beginning the “professional cycle”, which is, in turn, constituted by the advanced courses in physics. Saying in another way, it’s in this discipline that students have, usually, a more profound contact with Physics, inasmuch as this “professional cycle” is constituted by disciplines that are often offered only in Physics undergraduate, being many times absent in engineering and other scientific courses.

Therefore, the didactical approach to be applied in this discipline must be well designed, so as to allow the student to completely explore the richness of the subject to be worked, since that Classical Mechanics, together with Electromagnetic Theory, Statistical Mechanics and Quantum Mechanics, constitute the main axis of knowledge in Physics, essential for whatever be the area that student decide to follow in Graduation.

Considering this outlook, it was elaborated a strategy of class with the purpose of improving the teaching of mechanical oscillations, using for that an application in Condensed Matter Physics, more specifically to atomic force microscopy technique.

In this article we will cover the fundamental concepts regarding to the motion equation of the atomic force microscope (AFM) cantilever in chapter II and, following, we will discuss the solutions of the equation for each of different oscillation cases.

2. THE ATOMIC FORCE MICROSCOPE

The scanning probe microscopy (SPM) is a family of microscopy techniques widely used nowadays in investigations in nanoscience and nanotechnology. These techniques consist in measuring the interaction intensity between a probe (Fig. 1) and a sample, in order to determine the variation in the tip-sample distance in each point of the sample surface. This means that the microscope works by setting up a matrix of points z(x,y), where z represents de tip-sample distance, and the coordinates x and y are made to be along the sample surface [2].

The collected data are used to construct a topography image (as mountains and valleys) of the surface, what is done through the conversion of the z values in pixels by an electronic module coupled to microscope, as shown in Fig. 2. The obtained images are a representation of the surface relief of the analyzed material, and give many important information about its structure. This allows the study of properties such as: roughness, adhesion, wettability, stiffness and mechanical resistance, to name few [4].

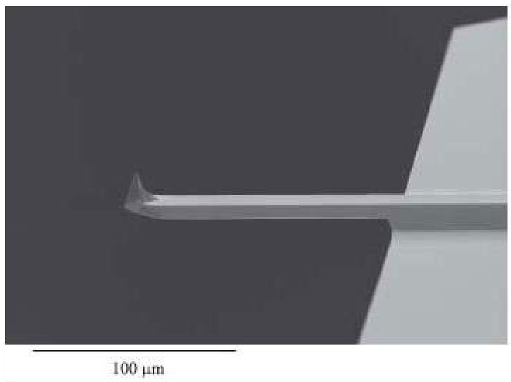

Fig. 1. Atomic force microscope cantilever, with the tip (probe) mounted in its end. Image obtained by scanning electron microscopy (SEM) [3]

In such precision measurements, whatever the data collection device, such as AFM cantilever or a quartz crystal as a sense element for some applications, the collected data are also used to construct a different type of shape working as a weighting function which very well compensates all disturbances. Besides the AFM, it is also used in porosity [5] or humidity [6] measurements. The collected data can be also used to construct different types of parallel curves, which compensates temperature influence. This was also reported for capacitance-frequency converters [7], used for the measurement of mechanical displacement, dielectric properties and density of liquids, small volumes or levels, pressure, flow and humidity, and for inductance-to-frequency converters [8], which are also used for measurement of mechanical displacement, as well as for nanopositioning, eccentric motion and strain sensing.

AFM is a member of the SPM Family. In this variety of microscope, the probe, also called tip, is mounted in a stem named cantilever. The variations in the intensity of the tip-sample interaction causes the cantilever to deflect proportionally in z direction, what is measured by an optical apparatus and saved in the computer (Fig. 3).

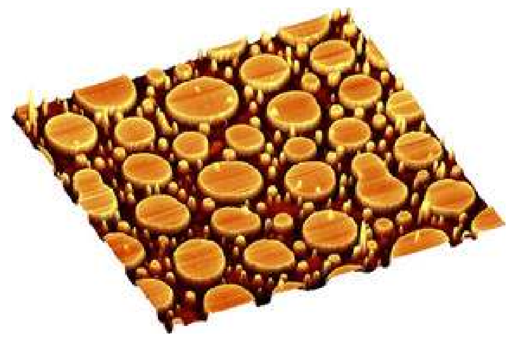

Fig. 2. AFM image of a PMMA-SBR (poly-methy-methacrylate – styrol-butadien-rubber) polymer blend, spin coated on glass. 20X20 µm scan, 30 nm topographic scale (height) [9]

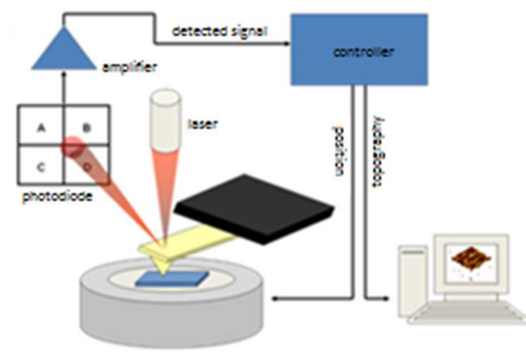

Fig. 3. AFM scheme. The cantilever deflections change the reflection angle of the laser during the scanning, what is detected by the photodiode. The controller receives this information and generates a signal to the piezoelectric ceramic, which moves to adjust the tip-sample distance. At each movement of the ceramic, the controller registers the data and generates the image of the surface

Between the AFM tip and the sample, as well as between any two other materials in contact or sufficiently close, there are intermolecular interaction forces, which belongs to a range of varieties but are called in general van der Waals interaction.

For a quantitative analysis of this interaction, one can derivate the total interaction through an integration process starting from the interaction between two single atoms, which in this case is well modeled by the Lennard-Jones potential (eq. 1) [10].

$$E = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \quad (1)$$

Where ε is a constant characteristic of the two atoms, σ is the average diameter of the two atoms and r is the distance between the atom centers. This potential has an attractive term, which dominates while the tip is far from the sample, and a repulsive term, that dominates when the tip approaches or touches the sample (Fig. 4).

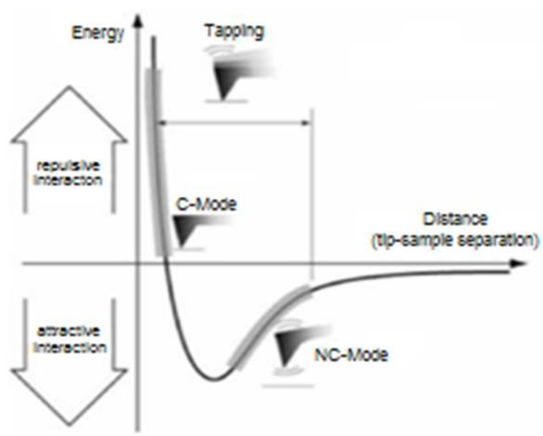

Fig. 4. Interaction Potential between two single atoms. The force (F) between the atoms can be obtained by the derivative of this potential. The distance for which the energy is minimum is called equilibrium point (F=0), and corresponds to a typical chemical bond [11]

The AFM device can be operated in three different modes. The first one is called contact mode, wherein the tip remains continuously in the repulsive forces zone. Another operation mode is the non-contact mode, wherein the tip doesn't touch the sample during the whole scanning and the existing interactions are essentially attractive.

There is also the intermittent contact mode, or tapping mode, in which an excitation signal is applied to the cantilever, causing it to vibrate, touching the sample surface once per oscillation. This mode is called intermittent because the interaction alternates between attractive and repulsive.

Some models have been proposed for the motion of Cantilever, considering the different modes of operation, and in the most general one it has been modeled as a Non-linear driven harmonic oscillator with damping [4], whose movement is governed by the following equation:

$$m\frac{d^2z}{dt^2} = -kz - \frac{m\omega_0}{Q}\frac{dz}{dt} + F_0\cos(\omega t) + F(z, z_c) \quad (2)$$

In the above equation, m, k, ω_0 and Q represent respectively, the mass, the spring constant, the resonant frequency and quality factor of the stem, determined from its dimensions and from its constituent material. The term $-kz$ is the elastic force and the term $-\frac{m\omega_0}{Q}\frac{dz}{dt}$ represents the environment damping. The sinusoidal term $F_0\cos(\omega t)$ is the excitation signal applied to the cantilever and $F(z, z_c)$ is the interaction between tip and sample around the equilibrium point z_c.

Some different models have been also proposed to determine $F(z, z_c)$. The difference between the three modes of operation relies on the form of the term $F(z, z_c)$, which is different for the two regions separated by the distance a_0 (Fig. 4). For the non-contact zone, the form of $F(z, z_c)$ is obtained considering the van der Waals interactions. Therefore, the interaction of the tip, mounted on the cantilever, with the surface can be approximated by those of a sphere, attached to a spring, interacting with a flat surface, as indicated in Fig. 4. In the contact zone the repulsive force is determined by the indentation force, obtained from Hertz model [12]:

$$F(z, z_c) = -\frac{AR}{6(z_c+z)^2}\, z_c + z \le a_0 \quad (3a)$$

$$F(z, z_c) = -\frac{A}{6a_0} + \frac{4ER^{1/2}}{3-3v^2}(a_0 - z - z_c)^{3/2}\, z_c + z \ge a_0 \quad (3b)$$

Where R is the tip radius, A is the Hamaker constant, and E and v are the Young modulus and Poisson coefficient of the sample, respectively.

Eq. (3a) applies for contact mode, regarding the fact that the sinusoidal term in Eq. (2) is absent, and (3b) for non-contact mode. In tapping mode, $F(z, z_c)$ alternates between (3a) and (3b) for each oscillation.

However, beyond the elastic deformations, the model used to explain the penetration of the tip in the sample (a phenomenon called nanoindentation) should take in consideration also the plastic deformations, that is, the permanent deformations on the material. Due to this is necessary to include some aspects of tensor analysis, like the generalized Hooke law equation and the deformation tensor [13].

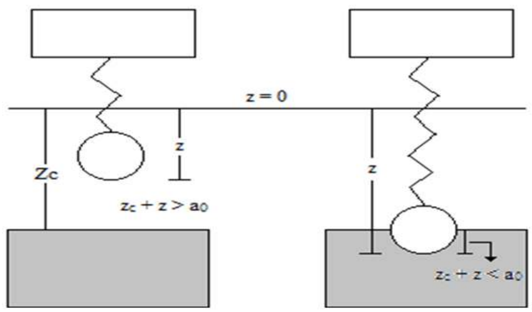

Fig. 5. Tip-sample interaction in tapping mode. In the left, the tip is away from the sample. In the right, the tip elastically indents the surface

In any way, the existence of the term $F(z, z_c)$ in equation (2) is responsible for inserting other vibration modes of the cantilever, compared to the one it would have if the above term do not exist [14]. Nevertheless, the solution of this equation in the conditions determined above occurs only by numerical methods. For the purpose of this article, the interaction between tip and sample is neglected and the differential equations are solved only for the first normal mode of vibration of the cantilever.

3. DIFFERENTIAL EQUATIONS

Neglecting the interaction between probe and sample, eq. (2) becomes:

$$m\frac{d^2z}{dt^2} = -kz - \frac{m\omega_0}{Q}\frac{dz}{dt} + F_0\cos(\omega t) \quad (4)$$

In the following sections are shown the methods to determine the general solution of this equation considering different physical situations.

3.1 1st Case: Dynamic Equilibrium

In this case, the resulting force on the cantilever is null, therefore eq. (4) takes the form:

$$kz + \frac{m\omega_0}{Q}\frac{dz}{dt} = F_0\cos(\omega t) \quad (5)$$

The solution of this equation can be obtained by the method of the integration factor. This method applies for differential equations of the form:

$$\frac{dy}{dx} + ay = f(x)$$

And consists in multiplying the whole equation by a function $h(x)$, so that the left side of the equation becomes the derivative of a product. This integration factor is given by e^{at}. Rewriting (5) we obtain:

$$\frac{dz}{dt} + \frac{kQ}{m\omega_0}z = \frac{F_0Q}{m\omega_0}\cos(\omega t)$$

Multiplying by $e^{\frac{kQ}{m\omega_0}t}$:

$$\frac{dz}{dt}e^{\frac{kQ}{m\omega_0}t} + \frac{kQ}{m\omega_0}e^{\frac{kQ}{m\omega_0}t}z = \frac{F_0Q}{m\omega_0}\cos(\omega t)\,e^{\frac{kQ}{m\omega_0}t}$$

$$\frac{d}{dt}\left(ze^{\frac{kQ}{m\omega_0}t}\right) = \frac{F_0Q}{m\omega_0}\cos(\omega t)\,e^{\frac{kQ}{m\omega_0}t}$$

Integrating in function of time:

$$\int d\left(ze^{\frac{kQ}{m\omega_0}t}\right)dt = \frac{F_0Q}{m\omega_0}\int\cos(\omega t)\,e^{\frac{kQ}{m\omega_0}t}dt \quad ze^{\frac{kQ}{m\omega_0}t} = \frac{F_0Q}{m\omega_0}\int\cos(\omega t)\,e^{\frac{kQ}{m\omega_0}t}dt \quad (6)$$

Integrating by parts the integral in the right side (6), we have:

$$\int\cos(\omega t)\,e^{\frac{kQ}{m\omega_0}t}dt = \left(\frac{k^2Q^2}{k^2Q^2+m^2\omega^2{\omega_0}^2}\right)\left[\cos\omega t + \frac{m\omega\omega_0}{kQ}\sin\omega t\right]\frac{m\omega_0}{kQ}e^{\frac{kQ}{m\omega_0}t}$$

Substituting in (6), we have

$$z = \frac{F_0kQ^2}{k^2Q^2+m^2\omega^2{\omega_0}^2}\cos\omega t + \frac{F_0Qm\omega\omega_0}{k^2Q^2+m^2\omega^2{\omega_0}^2}\sin\omega t \quad (7)$$

This solution shows that the cantilever behaves like an harmonic oscillator, once that its motion equation is a linear combination of harmonic terms, that is, sinusoidal terms.

As there is no dissipative term, or in other terms, the amplitude of the movement is constant, the energy of the movement is also constant. The total energy of the movement, in any instant, is calculated by the sum of kinetic and potential energy:

$$E = \frac{m\dot{z}^2}{2} + \frac{kz^2}{2} \qquad (8)$$

Then, from (7) and (8), one have:

$$E = \frac{k}{2}\left[\frac{F_0{}^2k^2Q^4}{(k^2Q^2+m^2\omega^2\omega_0{}^2)^2} + \frac{F_0{}^2Q^2m^2\omega^2\omega_0{}^2}{(k^2Q^2+m^2\omega^2\omega_0{}^2)^2}\right] \qquad (9)$$

3.2 2nd Case: Simple Oscillations

In this case, cantilever is considered to not being under damping forces nor external forces, so that the elastic force is the resulting force. Thus, eq. (4) takes the following form:

$$m\frac{d^2z}{dt^2} = -kz$$

Or also, considering that $\omega_0{}^2 = k/m$ and rewriting the above equation,

$$\frac{d^2z}{dt^2} + \omega_0{}^2 z = 0 \qquad (10)$$

To get the general solution of the equation (10), one must find a function of the form e^{rt} inasmuch as the second derivative reproduces the original function. Then, supposing a solution of this type, it leads to the following result:

$$z = Ce^{rt}; \quad \frac{dz}{dt} = Cre^{rt}; \quad \frac{d^2z}{dt^2} = Cr^2e^{rt}$$

What, substituting in eq. (10), leads to:

$$r^2 + \omega_0{}^2 = 0 \quad r = \pm i\omega_0 \qquad (11)$$

The result expressed in (11) shows that exist two possible solutions for this equation ($z_1 = A_1e^{i\omega_0 t}$ and $z_2 = A_2e^{-i\omega_0 t}$). It's convenient to express a general solution as a linear combination of these two simplest solutions, in order to the function thus obtained is also a solution of (10).

$$z = A_1e^{i\omega_0 t} + A_2e^{-i\omega_0 t}$$
$$z = A_1(\cos\omega_0 t + i\sin\omega_0 t) + A_2(\cos\omega_0 t - i\sin\omega_0 t)$$
$$z = (A_1 + A_2)\cos\omega_0 t + i(A_1 - A_2)\sin\omega_0 t \qquad (12)$$

If we make a convenient substitution, we can express this solution in form of an amplitude A and a phase φ [15], in the following way:

$$A_1 + A_2 = A\cos\varphi \qquad (13a)$$

$$A_1 - A_2 = iA\sin\varphi \qquad (13b)$$

Substituting (13) in (12):

$$z = \mathrm{A}\cos\varphi\cos\omega_0 t - \mathrm{A}\sin\varphi\sin\omega_0 t$$
$$z = \mathrm{A}\cos(\omega_0 t + \varphi) \qquad (14a)$$

Equation (14a) is the general solution of (10). The constants A and φ should be found by the boundary conditions, which are usually the position ($z_0 = z(0)$) and velocity ($\dot{z_o} = \frac{dz}{dt}(0)$) values in the initial instant $t = 0$. Using the boundary conditions in the general solution, it's found the following values for the constants:

$$A = \frac{z_0}{\cos\varphi} \qquad (14b)$$

$$\varphi = -\tan^{-1}\left(\frac{\dot{z_0}}{z_0\omega_0}\right) \qquad (14c)$$

Similarly to the first case, this movement undergoes no energy dissipation, and the total energy in this case is, from (8):

$$E = \frac{kz_0{}^2}{2} + \frac{m\dot{z_0}{}^2}{2} \qquad (15)$$

3.3 3rd Case: Damped Oscillations

In this case, the cantilever motion is considered to be damped by the environment, which may be air, water, or any other fluid. In this case, eq. (4) assumes the form:

$$m\frac{d^2z}{dt^2} = -kz - \frac{m\omega_0}{Q}\frac{dz}{dt} \qquad (16)$$

Or even

$$\frac{d^2z}{dt^2} + \frac{\omega_0}{Q}\frac{dz}{dt} + \omega^2 z = 0$$

The procedure is performed supposing a solution of the same form as in the previous situation, obtaining thus:

$$r^2 + \frac{\omega_0}{Q}r + \omega^2 = 0$$
$$r = -\frac{\omega_0}{2Q} \pm \left(\frac{\omega_0{}^2}{4Q^2} - \omega^2\right)^{1/2}$$
$$r = -\frac{\omega_0}{2Q} \pm \frac{1}{2Q}(\omega_0{}^2 - 4Q^2\omega^2)^{1/2}$$

Making the linear combination of the solutions:

$$z = e^{-\frac{\omega_0}{2Q}t}\left(\begin{array}{c} C_1e^{\left(\frac{1}{2Q}(\omega_0{}^2-4Q^2\omega^2)^{1/2}\right)t} \\ +C_2e^{-\left(\frac{1}{2Q}(\omega_0{}^2-4Q^2\omega^2)^{1/2}\right)t}\end{array}\right) \qquad (17)$$

One must consider three different possibilities as to the term within the root [15,16].

3.3.1 Overdamped oscillations

If $\omega_0 > 4Q^2\omega^2$, so the tem within the root is positive and the root is a real number, then it's said in this case that the motion of the cantilever is overdamped, and the oscillator reaches, without oscillating, an stability position determined by C_1 and C_2.

Considering the boundary conditions ($z_0 = z(0)$ and $\dot{z_o} = \frac{dz}{dt}(0)$), it's possible to determine the values of C_1 and C_2:

$$C_1 = \frac{\left(\omega_0 + (\omega_0{}^2 - 4Q^2\omega^2)^{1/2}\right)z_0 + 2Q\dot{z_o}}{2(\omega_0{}^2 - 4Q^2\omega^2)^{1/2}} \quad (18)$$

$$C_2 = \frac{\left(-\omega_0 + (\omega_0{}^2 - 4Q^2\omega^2)^{1/2}\right)z_0 - 2Q\dot{z_o}}{2(\omega_0{}^2 - 4Q^2\omega^2)^{1/2}} \quad (19)$$

3.3.2 Critically damped oscillations

Another possibility happens when $\omega_0{}^2 = 4Q^2\ \omega^2$. In this hypotheses the term inside the root is null and the general solution of (16) is simply:

$$z = Ce^{-\frac{\omega_0}{2Q}t}$$

This is the faster amplitude decay the object can execute without oscillating, and is called critically damped motion. Through the initial value $z_0 = z(0)$, it's easy to observe that $C = z_0$. So the solution for this case is:

$$z = z_0 e^{-\frac{\omega_0}{2Q}t} \quad (20)$$

3.3.3 Underdamped oscillations

The third case happens when $\omega_0{}^2 < 4Q^2\omega^2$. In this case the term inside the root is negative and the root is a complex number. We call this movement underdamped, because the amplitude decay occurs gradually as the cantilever oscillates. Then, Eq. (17) now becomes:

$$z = e^{-\frac{\omega_0}{2Q}t}\left(C_1 e^{i\left(\frac{1}{2Q}(\omega_0{}^2 - 4Q^2\omega^2)^{1/2}\right)t} + C_2 e^{-i\left(\frac{1}{2Q}(\omega_0{}^2 - 4Q^2\omega^2)^{1/2}\right)t}\right)$$

And developing the algebra in the same way that for the 2nd case, one finally obtains:

$$z = Ce^{-\frac{\omega_0}{2Q}t}\cos\left(\frac{1}{2}(4Q^2\omega^2 - \omega_0{}^2)^{1/2}t + \varphi\right) \quad (21)$$

Considering the term within the root in (17), this is, experimentally, the typical situation of the operation of the AFM cantilever. This is because we have typically $Q^2 \approx 10^4$, therefore, $\omega_0{}^2 - 4Q^2\omega^2 \approx -4Q^2\ \omega^2$ and the dissipation is underdamped [16].

The exponential term in Eq. (21) is called envelope, since it determines the decay in the oscillation amplitude. From the boundary conditions ($z_0 = z(0)$;$\dot{z_o} = \frac{dz}{dt}(0)$), it's possible to determine the values of C and φ:

$$C = \frac{z_0}{\cos\varphi} \quad (22)$$

$$\varphi = \tan^{-1}\left(\frac{-2Q\dot{z_o} - z_0\omega_0}{z_0(4Q^2\omega^2 - \omega_0{}^2)^{1/2}}\right) \quad (23)$$

The total energy in this case is:

$$E = \frac{m z_0{}^2}{4Q\cos^2\varphi} e^{\frac{-\omega_0}{Q}t}\left[\frac{\omega_0{}^2 + 2Q^2\omega^2}{2Q}\cos(\omega t + \varphi) + (\omega\omega_0 + 2Q\omega^2)\sin(\omega t + \varphi)\right] \quad (24)$$

The exponential term represents the energy dissipation, so the energy goes to zero as time passes.

3.4 4th Case: Forced Oscillations

When a force is applied to the cantilever, through an excitation signal, and disregarding the environment damping, eq. (4) turns

$$m\frac{d^2z}{dt^2} = -kz + F_0\cos(\omega t) \quad (25)$$

This equation can be written in the following form:

$$\frac{d^2z}{dt^2} + \omega_0{}^2 z = \frac{F_0}{m}\cos(\omega t) \quad (26)$$

The general solution for an equation of the form of (26) are given by the sum of a complementary solution (homogeneous solution) and a particular solution (non-homogeneous solution), without loss of generality.

$$z = z_c + z_p$$

The complementary solution is given by Eqs. (14). To find the particular solution, we suppose an harmonic function which oscillates in phase ($\varphi = 0$) and with the same frequency ω of the source:

$$z_p = B\cos(\omega t) \tag{27}$$

Substituting (27) in (16), one finds

$$-B\omega^2\cos(\omega t) + B{\omega_0}^2\cos(\omega t) = \frac{F_0}{m}\cos(\omega t)$$
$$B = \frac{F_0}{m({\omega_0}^2 - \omega^2)} \tag{28}$$

So, the vibration of the cantilever is composed of linear combinations of normal modes, but now each small vibration occurs with the same frequency of the applied force.

As a consequence of the denominator of eq. (28), the closer ω is from ω_0 the more intense will be the excitation on this vibration mode. Apparently, Eq. (28) predicts an infinite amplitude when $\omega = \omega_0$, but this is not physically plausible, since the theory behind this equation is grounded for small oscillations around the equilibrium position, so that (28) is not still valid if the amplitude becomes much larger.

Therefore, the general solution is given by:

$$z(t) = A\cos(\omega_0 t + \varphi) + \frac{F_0}{m({\omega_0}^2 - \omega^2)}\cos(\omega t) \tag{29}$$

The constants A and φ should be determined through the boundary conditions, what leads to:

$$A = \frac{-\dot{z_o}}{\omega_0\sin\varphi} \tag{30}$$

$$\varphi = \cot^{-1}\left(\frac{\omega_0 F_0}{\dot{z_o} m({\omega_0}^2 - \omega^2)} - \frac{\omega_0 z_o}{\dot{z_o}}\right) \tag{31}$$

If $\omega = \omega_0$ so the supposition of eq. (27) is a solution of the homogeneous problem, and must be substituted by another one of the form

$$z_p = Bt\cos(\omega_0 t)$$

This term presents a linear growth in amplitude as it oscillates, what characterizes a phenomenon called resonance. The oscillator, thereby, continually receives energy provided by the external force.

3.5 5th Case: Damped and Forced Oscillations

This is the most general case in our description, in which Eq. (4) undergoes no change. The frequency of a forced oscillator is determined by the frequency of the external force and not by the resonant frequency [17]. To study what would be the general solution in this case, eq. (4) is written in the following manner:

$$\frac{d^2z}{dt^2} + \frac{\omega}{Q}\frac{dz}{dt} + \omega^2 z = \frac{F_0}{m}\cos(\omega t) \tag{32}$$

As for Eq. (19), the general solution for Eq. (32) involves a complementary and a particular solution. For the same reason of the previous case, the complementary solution is the solution of the homogeneous equation, Eq. (21). To find the particular solution of (32), it is assumed a function of the form:

$$z_p(t) = D_1\cos(\omega t) + D_2\sin(\omega t) \tag{33}$$

Substituting (33) in (32), and solving the linear system, one obtains $D_1 = 0$ and $D_2 = F_0Q/\omega^2$. Then the particular solution is:

$$z_p = \frac{F_0 Q}{\omega^2}\sin(\omega t)$$

And, in turn, the general solution is:

$$z(t) = Ce^{-\frac{\omega_0}{2Q}t}\cos(\omega t + \varphi) + \frac{F_0 Q}{\omega^2}\sin(\omega t) \tag{34}$$

From the initial conditions, the constants for this case are given by (30) and (31), once that changes took place only in the particular solution.

4. CONCLUSION

We purposed a didactic mechanism for the teaching of the physical and mathematical concepts with respect to the mechanical oscillations, which is commonly addressed in undergraduate courses in Physics, in the chair of Classical Mechanics or correlated ones.

By applying the differential equations to a recent and instigating research area, the subject becomes better understood by students, gaining great results of learning.

This also evidences the practical character of mathematics that is studied in undergraduation, associating it to physical concepts and making the learning to be more significant.

This also contributes to the professional formation of the undergraduate student, since the discipline of classical mechanics is essential for the subsequent other disciplines of undergraduation and, eventually, graduation.

This purpose needs, however, further work to confirm its educational potential by applying to an undergraduation class.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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