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A New Common Tripled Fixed Point Result in Two Quasi-Partial b-metric Spaces

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Abstract

The aim of this paper is to prove some new common tripled fixed point theorems for mappings defined a set equipped with two quasi-partial b-metric spaces with the same coefficient s. Some examples are also given in support of our new results.

Keywords: Common tripled fixed point; tripled coincidence point; w-compatible mappings; quasi-partial metric space; quasi-partial b-metric space.

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1 Introduction and Preliminaries

The notion of partial metric spaces was introduced by Matthews [1] in 1994. He extended the Banach Contraction Principle from metric spaces to partial metric spaces. Several authors (for examples, [2], [3], [4], [5], [6], [7], [8] worked on this notion of partial metric spaces and obtained fixed point results for mappings satisfying different contractive conditions. Haghi et al. [9] showed in their interesting paper that some of fixed point theorems in partial metric spaces can be obtained from metric spaces.

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Karapinar et al. [10] introduced the concept of quasi-partial metric spaces and studied some fixed point problems on it.

The notion of partial metric space is given as follows:

Definition 1.1. (Matthews [1]) A partial metric on a nonempty set X is a functon $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(P_1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(P_2) $p(x,x) \leq p(x,y)$,

(P_3) $p(x,y) = p(y,x)$,

(P_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X. For a partial metric p on X, the function $d_p : X \times X \to \mathbb{R}^+$ defined by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y) \text{ is a metric on } X.$$

Karapinar et al. [10] gave the notion of quasi-partial metric spaces as follows.

Definition 1.2. (Karapinar et al. [10]) A quasi-partial metric on nonempty set X is a function $q : X \times X \to \mathbb{R}^+$ which satisfies:

(QPM_1) if $q(x,x) = q(x,y) = q(y,y)$, then $x = y$,

(QPM_2) $q(x,x) \leq q(x,y)$,

(QPM_3) $q(x,x) \leq q(y,x)$, and

(QPM_4) $q(x,y) + q(z,z) \leq q(x,z) + q(z,y)$ for all $x, y, z \in X$.

A quasi-partial metric space is a pair (X,q) such that X is a nonempty set and q is a quasi-partial metric on X.

Let q be a quasi-partial metric on set X. Then $d_q(x,y) = q(x,y) + q(y,x) - q(x,x) - q(y,y)$ is a metric on X.

Lemma 1.1. (Karapinar et al. [10]) *Let (X,q) be a quasi-partial metric space. Let (X,p_q) be the corresponding partial metric space, where $p_q(x,y) = 1/2[q(x,y) + q(y,x)]$ for all $x, y \in X$ is a partial metric on X, and let (X, d_{p_q}) be the corresponding metric space. Then following statements are equivalent*

(i) (X,q) *is complete,*

(ii) (X,p_q) *is complete,*

(iii) (X,d_{p_q}) *is complete.*

Moreover,

$$\begin{aligned}\lim_{n\to\infty} d_{p_q}(x,x_n) = 0 \Leftrightarrow p_q(x,x) &= \lim_{n\to\infty} p_q(x,x_n) = \lim_{n,m\to\infty} p_q(x_n,x_m) \\ \Leftrightarrow q(x,x) &= \lim_{n\to\infty} q(x,x_n) = \lim_{n,m\to\infty} q(x_n,x_m) \\ &= \lim_{n\to\infty} q(x_n,x) = \lim_{n,m\to\infty} q(x_m,x_n).\end{aligned}$$

In 1989, Bakhtin [11] introduced the concept of a b-metric space as a generalization of metric space which was further extended by Czerwik [12]. Later, Shukla [13] generalized both the concepts of b-metric and partial metric spaces by introducing the partial b-metric spaces.

Definition 1.3. (Shukla, [13]) A partial b-metric on a nonempty set X is a mapping $p_b : X \times X \to \mathbb{R}^+$ such that for some real number $s \geq 1$ and for all $x, y, z \in X$

($\mathrm{P_{b_1}}$) $x = y$ if and only if $p_b(x,x) = p_b(x,y) = p_b(y,y)$,

($\mathrm{P_{b_2}}$) $p_b(x,x) \leq p_b(x,y)$,

($\mathrm{P_{b_3}}$) $p_b(x,y) = p_b(y,x)$,

($\mathrm{P_{b_4}}$) $p_b(x,y) \leq s[p_b(x,z) + p_b(z,y)] - p_b(z,z)$.

A partial b-metric space is a pair (X, p_b) such that X is nonempty set and p_b is a partial b-metric on X. The number s is called the coefficient of (X, p_b).

The notion of quasi-partial b-metric space was introduced by Gupta and Gautam [15] where fixed point theorem was proved on it. Later this study was extended to coupled fixed point theorems on quasi-partial b-metric spaces in [14].

Definition 1.4. (Gupta and Gautam [15]) A quasi-partial b-metric on a nonempty set X is a mapping $qp_b : X \times X \to \mathbb{R}^+$ such that for some real number $s \geq 1$ and for all $x, y, z \in X$

($\mathrm{QP_{b_1}}$) $qp_b(x,x) = qp_b(x,y) = qp_b(y,y) \Rightarrow x = y$,

($\mathrm{QP_{b_2}}$) $qp_b(x,x) \leq qp_b(x,y)$,

($\mathrm{QP_{b_3}}$) $qp_b(x,x) \leq qp_b(y,x)$,

($\mathrm{QP_{b_4}}$) $qp_b(x,y) \leq s[qp_b(x,z) + qp_b(z,y)] - qp_b(z,z)$.

A quasi-partial b-metric space is a pair (X, qp_b) such, that X is a nonempty set and qp_b is a quasi-partial b-metric on X. The number s is called the coefficient of (X, qp_b). Let qp_b be a quasi-partial b-metric on the set X.

Then $d_{qp_b}(x,y) = qp_b(x,y) + qp_b(y,x) - qp_b(x,x) - qp_b(y,y)$ is a b-metric on X.

Lemma 1.2. (Gupta and Gautam [15]) *Every partial b-metric space is a quasi-partial b-metric space. But the converse need not be true.*

Lemma 1.3. (Gupta and Gautam [15]) *Let (X, qp_b) be a quasi-partial b-metric space. Then the following hold*

(A) *If $qp_b(x,y) = 0$ then $x = y$,*

(B) *If $x \neq y$, then $qp_b(x,y) > 0$ and $qp_b(y,x) > 0$.*

The proof is similar to the proof for the case of quasi-partial metric space ([10]).

Definition 1.5. (Gupta and Gautam [15]) Let (X, qp_b) be a quasi-partial b-metric space. Then

(i) a sequence $\{x_n\} \subset X$ converges to $x \in X$ if and only if

$$qp_b(x,x) = \lim_{n\to\infty} qp_b(x,x_n) = \lim_{n\to\infty} qp_b(x_n,x).$$

(ii) a sequence $\{x_n\} \subset X$ is called a Cauchy sequence if and only if

$$\lim_{n,m\to\infty} qp_b(x_n,x_m) \quad \text{and} \quad \lim_{n,m\to\infty} qp_b(x_m,x_n) \text{ exist (and are finite).}$$

(iii) The quasi partial b-metric space (X, qp_b) is said to be complete if every Cauchy sequence $\{x_n\} \subset X$ converges with respect to τ_{qp_b} to a point $x \in X$ such that $qp_b(x,x) = \lim_{n,m\to\infty} qp_b(x_m,x_n)$ $= \lim_{n,m\to\infty} qp_b(x_n,x_m)$.

Lemma 1.4. (Gupta and Gautam [15]) *Let (X, qp_b) be a quasi-partial b-metric space and (X, d_{qp_b}) be the corresponding b-metric space. Then (X, d_{qp_b}) is complete if (X, qp_b) is complete.*

Bhaskar and Lakshmikantham [16] introduced the concept of coupled fixed point and studied some coupled fixed point theorems. Later, Lakshmikantham and Ćirć [17] introduced the notion of a coupled coincidence point of mappings. For some works on a coupled fixed point, we refer to [18], [19].

For simplicity, we denote from now on $\underbrace{X \times X \times \cdots \times X}_{k \text{ terms}}$ by X^k where $k \in \mathbb{N}$ and X is a nonempty set. We begin with the following:

Definition 1.6. (Bhaskar and Lakshmikantham [16]) An element $(x, y) \in X^2$ is called a coupled fixed point of the mapping $F : X^2 \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 1.7. (Lakshmikantham and Ćirć [17]) An element $(x, y) \in X^2$ is called

(i) a *coupled coincidence point* of the mapping $F : X^2 \to X$ and $g : X \to X$ if $F(x, y) = gx$ and $F(y, x) = gy$, and (gx, gy) is called a *coupled point of coincidence*;

(ii) a *common coupled fixed point* of mappings $F : X^2 \to X$ and $g : X \to X$ if $F(x, y) = gx = x$ and $F(y, x) = gy = y$.

Definition 1.8. (Abbas et al. [20]) The mappings $F : X^2 \to X$ and $g : X \to X$ are called *w-compatible* if $gF(x, y) = F(gx, gy)$ whenever $F(x, y) = gx$ and $F(y, x) = gy$.

In 2010, Samet and Vetro [21] introduced a fixed point of order $N \geq 3$. In particular, for $N = 3$, we have the following definition.

Definition 1.9. (Samet and Vetro [21]) An element $(x, y, z) \in X^3$ is called a *tripled fixed point* of a given mapping $F : X^3 \to X$ if $F(x, y, z) = x$, $F(y, z, x) = y$, and $F(z, x, y) = z$.

Recently, Aydi and Abbas [22] obtained some tripled co-incidence and fixed point results in partial metric space.

Berinde and Borcut [23] defined differently the notion of tripled fixed point in the case of ordered sets in order to keep true the mixed monotone property. For more literature on tripled fixed points, see [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37] and [38].

Definition 1.10. (Aydi et al. [39]) An element $(x, y, z) \in X^3$ is called

(i) a *tripled coincidence point* of mappings $F : X^3 \to X$ and $g : X \to X$ if $F(x, y, z) = gx$, $F(y, z, x) = gy$, and $F(z, x, y) = gz$.

In this case (gx, gy, gz) is called a *tripled point of coincidence*;

(ii) a *common tripled fixed point* of mappings $F : X^3 \to X$ and $g : X \to X$ if $F(x, y, z) = gx = x$, $F(y, z, x) = gy = y$, and $F(z, x, y) = gz = z$.

Definition 1.11. (Aydi et al. [20]) The mappings $F : X^3 \to X$ and $g : X \to X$ are called w-compatible if $gF(x, y, z) = F(gx, gy, gz)$ whenever $F(x, y, z) = gx$, $F(y, z, x) = gy$, and $F(z, x, y) = gz$.

Shatanawi and Pitea [40] obtained some common coupled fixed point results for a pair of mappings in quasi-partial metric space. Motivated by their work we have studied some coupled fixed point theorems in quasi-partial b-metric space.

Theorem 1.5. (Gupta and Gautam [15])*Let (X, qp_b) be a quasi-partial b-metric space, $g : X \to X$ and $F : X \times X \to X$ be two mappings. Suppose that there exist $k_1, k_2, k_3 \in [0, 1)$ with $k_1+k_2+k_3 < 1$ and $k_3 < \frac{1}{s}$ where $s \geq 1$ such that the condition*

$$\begin{aligned} & qp_b(F(x, y)\, F(u, v)) + qp_b(F(y, x), F(v, u)) \\ & \quad \leq k_1[qp_b(gx, gu) + qp_b(gy, gv)] + k_2[qp_b(gx, F(x, y)) + qp_b(gy, F(y, x))] \\ & \quad\quad + k_3[qp_b(gu, F(u, v)) + qp_b(gv, F(v, u))] \end{aligned} \tag{1.1}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X \times X) \subseteq g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_b.*

Then the mappings F and g have a coupled coincidence point (x, y) satisfying $gx = F(x, y) = F(y, x) = gy$. Moreover, if F and g are w-compatible,then F and g have a unique common fixed point of the form (x, x).

The aim of this article is to prove some new common tripled fixed point theorems for mappings defined on a set equipped with two quasi-partial b-metric spaces. In this manuscript, we generalize, improve, enrich and extend the above coupled common fixed point results. Some examples are given to illustrate our results.

2 The Main Results

Theorem 2.1. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X with same coefficient $s \geq 1$ and $qp_{b_2}(x, y) \leq qp_{b_1}(x, y)$, for all $x, y \in X$, and let $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4 and k_5 in $[0, 1)$ with*

$$k_1 + k_2 + k_3 + 2sk_4 + k_5 < \frac{1}{s} \tag{2.1}$$

such that the condition

$$\begin{aligned} & qp_{b_1}(F(x, y, z), F(u, v, w)) + qp_{b_1}(F(y, z, x), F(v, w, u)) + qp_{b_1}(F(z, x, y), F(w, u, v)) \\ & \leq k_1[qp_{b_2}(gx, gu) + qp_{b_2}(gy, gv) + qp_{b_2}(gz, gw)] \\ & \quad + k_2[qp_{b_2}(gx, F(x, y, z)) + qp_{b_2}(gy, F(y, z, x)) + qp_{b_2}(gz, F(z, x, y))] \\ & \quad + k_3[qp_{b_2}(gu, F(u, v, w)) + qp_{b_2}(gv, F(v, w, u)) + qp_{b_2}(gw, F(w, u, v))] \\ & \quad + k_4[qp_{b_2}(gx, F(u, v, w)) + qp_{b_2}(gy, F(v, w, u)) + qp_{b_2}(gz, F(w, u, v))] \\ & \quad + k_5[qp_{b_2}(gu, F(x, y, z)) + qp_{b_2}(gv, F(y, z, x)) + qp_{b_2}(gw, F(z, x, y))] \end{aligned} \tag{2.2}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X^3) \subset g(X)$;*

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.*

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Proof. Let $x_0, y_0, z_0 \in X$. Since $F(X^3) \subset g(X)$, we can choose $x_1, y_1, z_1 \in X$ such that $gx_1 = F(x_0, y_0, z_0)$, $gy_1 = F(y_0, z_0, x_0)$ and $gz_1 = F(z_0, x_0, y_0)$. Similarly, we can choose $x_2, y_2, z_2 \in X$ such that $gx_2 = F(x_1, y_1, z_1)$, $gy_2 = F(y_1, z_1, x_1)$, and $gz_2 = F(z_1, x_1, y_1)$. Continuing in this way we construct three sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ in X such that

$$gx_{n+1} = F(x_n, y_n, z_n), \quad gy_{n+1} = F(y_n, z_n, x_n) \text{ and } gz_{n+1} = F(z_n, x_n, y_n), \quad \forall\, n \geq 0. \tag{2.3}$$

It follows from (2.2), (2.3), ($\mathrm{QP_{b_2}}$), and ($\mathrm{QP_{b_4}}$) that

$$\begin{aligned}
& qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1}) \\
&= qp_{b_1}(F(x_{n-1}, y_{n-1}, z_{n-1}), F(x_n, y_n, z_n)) + qp_{b_1}(F(y_{n-1}, z_{n-1}, x_{n-1}), F(y_n, z_n, x_n)) \\
&\quad + qp_{b_1}(F(z_{n-1}, x_{n-1}, y_{n-1}), F(z_n, x_n, y_n)) \\
&\leq k_1[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n) + qp_{b_2}(gz_{n-1}, gz_n)] \\
&\qquad + k_2[qp_{b_2}(gx_{n-1}, F(x_{n-1}, y_{n-1}, z_{n-1})) + qp_{b_2}(gy_{n-1}, F(y_{n-1}, z_{n-1}, x_{n-1})) \\
&\qquad + qp_{b_2}(gz_{n-1}, F(z_{n-1}, x_{n-1}, y_{n-1}))] \\
&\qquad + k_3[qp_{b_2}(gx_n, F(x_n, y_n, z_n)) + qp_{b_2}(gy_n, F(y_n, z_n, x_n)) + qp_{b_2}(gz_n, F(z_n, x_n, y_n))] \\
&\qquad + k_4[qp_{b_2}(gx_{n-1}, F(x_n, y_n, z_n)) + qp_{b_2}(gy_{n-1}, F(y_n, z_n, x_n)) + qp_{b_2}(gz_{n-1}, F(z_n, x_n, y_n)))] \\
&\qquad + k_5[qp_{b_2}(gx_n, F(x_{n-1}, y_{n-1}, z_{n-1})) + qp_{b_2}(gy_n, F(y_{n-1}, z_{n-1}, x_{n-1})) \\
&\qquad + qp_{b_2}(gz_n, F(z_{n-1}, x_{n-1}, y_{n-1}))] \\
&= (k_1 + k_2)[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n) + qp_{b_2}(gz_{n-1}, gz_n)] \\
&\qquad + k_3[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1}) + qp_{b_2}(gz_n, gz_{n+1})] \\
&\qquad + k_4[qp_{b_2}(gx_{n-1}, gx_{n+1}) + qp_{b_2}(gy_{n-1}, gy_{n+1}) + qp_{b_2}(gz_{n-1}, gz_{n+1})] \\
&\qquad + k_5[qp_{b_2}(gx_n, gx_n) + qp_{b_2}(gy_n, gy_n) + qp_{b_2}(gz_n, gz_n)] \\
&\leq (k_1 + k_2)[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n) + qp_{b_2}(gz_{n-1}, gz_n)] \\
&\qquad + k_3[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1}) + qp_{b_2}(gz_n, gz_{n+1})] \\
&\qquad + k_4[s\{qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gx_n, gx_{n+1})\} - qp_{b_2}(gx_n, gx_n) \\
&\qquad + s\{qp_{b_2}(gy_{n-1}, gy_n) + qp_{b_2}(gy_n, gy_{n+1})\} - qp_{b_2}(gy_n, gy_n) \\
&\qquad + s\{qp_{b_2}(gz_{n-1}, gz_n) + qp_{b_2}(gz_n, gz_{n+1})\} - qp_{b_2}(gz_n, gz_n)] \\
&\qquad + k_5[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1}) + qp_{b_2}(gz_n, gz_{n+1})] \\
&\leq (k_1 + k_2 + sk_4)[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n) + qp_{b_2}(gz_{n-1}, gz_n)] \\
&\qquad + (k_3 + sk_4 + k_5)[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1}) + qp_{b_2}(gz_n, gz_{n+1})] \\
&\leq (k_1 + k_2 + sk_4)[qp_{b_1}(gx_{n-1}, gx_n) + qp_{b_1}(gy_{n-1}, gy_n) + qp_{b_1}(gz_{n-1}, gz_n)] \\
&\qquad + (k_3 + sk_4 + k_5)[qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1})]
\end{aligned}$$

which implies that

$$\begin{aligned}
& qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1}) \\
&\quad \leq \frac{k_1 + k_2 + sk_4}{1 - k_3 - sk_4 - k_5}[qp_{b_1}(gx_{n-1}, gx_n) + qp_{b_1}(gy_{n-1}, gy_n) + qp_{b_1}(gz_{n-1}, gz_n)].
\end{aligned} \tag{2.4}$$

Put $k = \dfrac{k_1 + k_2 + sk_4}{1 - k_3 - sk_4 - k_5}$. Obviously by (2.1) $0 \leq k \leq 1$. Repeating the above inequality (2.4) n times, we get

$$\begin{aligned}
& qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1})] \\
&\quad \leq k^n[qp_{b_1}(gx_0, gx_1) + qp_{b_1}(gy_0, gy_1) + qp_{b_1}(gz_0, gz_1)].
\end{aligned} \tag{2.5}$$

Next, we shall prove that $\{gx_n\}$, $\{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences in $g(X)$.

In fact, for each $n, m \in \mathbb{N}$, $m > n$, from $(\mathrm{QP}_{\mathrm{b}_4})$ and (2.5) we have

$$
\begin{aligned}
& qp_{b_1}(gx_n, gx_m) + qp_{b_1}(gy_n, gy_m) + qp_{b_1}(gz_n, gz_m) \\
& \leq \sum_{i=n}^{m-1} s^{m-i}[qp_{b_1}(gx_i, gx_{i+1}) + qp_{b_1}(gy_i, gy_{i+1}) + qp_{b_1}(gz_i, gz_{i+1})] \\
& \leq \sum_{i=n}^{m-1} s^{m-i}k^i[qp_{b_1}(gx_0, gx_1) + qp_{b_1}(gy_0, gy_1) + qp_{b_1}(gz_0, gz_1)] \\
& = \sum_{i=n}^{m-1} \left(\frac{k}{s}\right)^i \cdot s^m[qp_{b_1}(gx_0, gx_1) + qp_{b_1}(gy_0, gy_1) + qp_{b_1}(gz_0, gz_1)] \\
& \leq \sum_{i=n}^{\infty} \left(\frac{k}{s}\right)^i \cdot s^m[qp_{b_1}(gx_0, gx_1) + qp_{b_1}(gy_0, gy_1) + qp_{b_1}(gz_0, gz_1)] \\
& = \frac{\left(\frac{k}{s}\right)^n}{\left(1 - \frac{k}{s}\right)} \cdot s^m[qp_{b_1}(gx_0, gx_1) + qp_{b_1}(gy_0, gy_1) + qp_{b_1}(gz_0, gz_1)] \qquad (2.6)
\end{aligned}
$$

Since $\left(\frac{k}{s}\right) < 1$, letting $n \to \infty$ in (2.6) and holding m fixed, we get

$$\lim_{n\to\infty}[qp_{b_1}(gx_n, gx_m) + qp_{b_1}(gy_n, gy_m) + qp_{b_1}(gz_n, gz_m)] \leq 0\,.$$

But

$$\lim_{n\to\infty}[qp_{b_1}(gx_n, gx_m) + qp_{b_1}(gy_n, gy_m) + qp_{b_1}(gz_n, gz_m)] \geq 0\,.$$

This implies

$$\lim_{n\to\infty}[qp_{b_1}(gx_n, gx_m)] = \lim_{n\to\infty}[qp_{b_1}(gy_n, gy_m)] = \lim_{n\to\infty} qp_{b_1}(gz_n, gz_m) = 0\,.$$

Now letting $m \to +\infty$

$$\lim_{n,m\to\infty} qp_{b_1}(gx_n, gx_m) = \lim_{n,m\to\infty} qp_{b_1}(gy_n, gy_m) = \lim_{n,m\to\infty} qp_{b_1}(gz_n, gz_m) = 0\,. \qquad (2.7)$$

By similar arguments as above, we can show that

$$\lim_{n,m\to\infty} qp_{b_1}(gx_m, gx_n) = \lim_{n,m\to\infty} qp_{b_1}(gy_m, gy_n) = \lim_{n,m\to\infty} qp_{b_1}(gz_m, gz_n) = 0. \qquad (2.8)$$

Hence, $\{gx_n\}$, $\{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences in $(g(X), qp_{b_1})$.

Since $(g(X), qp_{b_1})$ is complete, there exist $gx, gy, gz \in g(X)$ such that $\{gx_n\}$, $\{gy_n\}$ and $\{gz_n\}$ converge to gx, gy and gz with respect to $\tau_{qp_{b_1}}$, where $\tau_{qp_{b_1}}$ is a quasi-partial b-metric topology, that is,

$$
\begin{aligned}
qp_{b_1}(gx, gx) &= \lim_{n\to\infty} qp_{b_1}(gx, gx_n) = \lim_{n\to\infty} qp_{b_1}(gx_n, gx) \\
&= \lim_{n,m\to\infty} qp_{b_1}(gx_m, gx_n) = \lim_{n,m\to\infty} qp_{b_1}(gx_n, gx_m), \qquad (2.9) \\
qp_{b_1}(gy, gy) &= \lim_{n\to\infty} qp_{b_1}(gy, gy_n) = \lim_{n\to\infty} qp_{b_1}(gy_n, gy) \\
&= \lim_{n,m\to\infty} qp_{b_1}(gy_m, gy_n) = \lim_{n,m\to\infty} qp_{b_1}(gy_n, gy_m) \qquad (2.10)
\end{aligned}
$$

and

$$\begin{aligned} qp_{b_1}(gz, gz) &= \lim_{n\to\infty} qp_{b_1}(gz, gz_n) = \lim_{n\to\infty} qp_{b_1}(gz_n, gz) \\ &= \lim_{n,m\to\infty} qp_{b_1}(gz_m, gz_n) = \lim_{n,m\to\infty} qp_{b_1}(gz_n, gz_m)\,. \end{aligned} \tag{2.11}$$

Combining (2.7)-(2.11), we have

$$\begin{aligned} qp_{b_1}(gx, gx) &= \lim_{n\to\infty} qp_{b_1}(gx, gx_n) = \lim_{n\to\infty} qp_{b_1}(gx_n, gx) \\ &= \lim_{n,m\to\infty} qp_{b_1}(gx_m, gx_n) = \lim_{n,m\to\infty} qp_{b_1}(gx_n, gx_m) = 0 \end{aligned} \tag{2.12}$$

and

$$\begin{aligned} qp_{b_1}(gy, gy) &= \lim_{n\to\infty} qp_{b_1}(gy, gy_n) = \lim_{n\to\infty} qp_{b_1}(gy_n, gy) \\ &= \lim_{n,m\to\infty} qp_{b_1}(gy_m, gy_n) = \lim_{n,m\to\infty} qp_{b_1}(gy_n, gy_m) = 0 \end{aligned} \tag{2.13}$$

and

$$\begin{aligned} qp_{b_1}(gz, gz) &= \lim_{n\to\infty} qp_{b_1}(gz, gz_n) = \lim_{n\to\infty} qp_{b_1}(gz_n, gz) \\ &= \lim_{n,m\to\infty} qp_{b_1}(gz_m, gz_n) = \lim_{n,m\to\infty} qp_{b_1}(gz_n, gz_m) = 0\,. \end{aligned} \tag{2.14}$$

On the other hand, by ($\mathrm{QP_{b_4}}$) we have

$$\begin{aligned} & qp_{b_1}(gx_{n+1}, F(x, y, z)) \\ & \quad \le s\{qp_{b_1}(gx_{n+1}, gx) + qp_{b_1}(gx, F(x, y, z))\} - qp_{b_1}(gx, gx) \\ & \quad \le s\{qp_{b_1}(gx_{n+1}, gx) + qp_{b_1}(gx, F(x, y, z))\} \\ & \quad \le s[qp_{b_1}(gx_{n+1}, gx) + s\{qp_{b_1}(gx, gx_{n+1}) + qp_{b_1}(gx_{n+1}, F(x, y, z))\} - qp_{b_1}(gx_{n+1}, gx_{n+1})] \\ & \quad \le s[qp_{b_1}(gx_{n+1}, gx)] + s^2[qp_{b_1}(gx, gx_{n+1})] + s^2[qp_{b_1}(gx_{n+1}, F(x, y, z,))]. \end{aligned}$$

Letting $n \to \infty$ in the above inequalities and using (2.12), we have

$$\frac{1}{s} qp_{b_1}(gx, F(x, y, z)) \le \lim_{n\to\infty} qp_{b_1}(gx_{n+1}, F(x, y, z)) \le s qp_{b_1}(gx, F(x, y, z))\,. \tag{2.15}$$

Similarly, using (2.13) and (2.14) we have

$$\frac{1}{s} qp_{b_1}(gy, F(y, z, x)) \le \lim_{n\to\infty} qp_{b_1}(gy_{n+1}, F(y, z, x)) \le s qp_{b_1}(gy, F(y, z, x)) \tag{2.16}$$

and

$$\frac{1}{s} qp_{b_1}(gz, F(z, x, y)) \le \lim_{n\to\infty} qp_{b_1}(gz_{n+1}, F(z, x, y)) \le s qp_{b_1}(gz, F(z, x, y))\,. \tag{2.17}$$

Now we prove that $F(x, y, z) = gx$, $F(y, z, x) = gy$ and $F(z, x, y) = gz$. It follows from (2.2) and

(2.3) that

$$\begin{aligned}
&qp_{b_1}(gx_{n+1}, F(x,y,z)) + qp_{b_1}(gy_{n+1}, F(y,z,x)) + qp_{b_1}(gz_{n+1}, F(z,x,y)) \\
&\quad = qp_{b_1}(F(x_n,y_n,z_n), F(x,y,z)) + qp_{b_1}(F(y_n,z_n,x_n), F(y,z,x)) + qp_{b_1}(F(z_n,x_n,y_n), F(z,x,y)) \\
&\quad \le k_1[qp_{b_2}(gx_n, gx) + qp_{b_2}(gy_n, gy) + qp_{b_2}(gz_n, gz)] \\
&\qquad + k_2[qp_{b_2}(gx_n, F(x_n,y_n,z_n)) + qp_{b_2}(gy_n, F(y_n,z_n,x_n)) + qp_{b_2}(gz_n, F(z_n,x_n,y_n))] \\
&\qquad + k_3[qp_{b_2}(gx, F(x,y,z)) + qp_{b_2}(gy, F(y,z,x)) + qp_{b_2}(gz, F(z,x,y))] \\
&\qquad + k_4[qp_{b_2}(gx_n, F(x,y,z)) + qp_{b_2}(gy_n, F(y,z,x)) + qp_{b_2}(gz_n, F(z,x,y))] \\
&\qquad + k_5[qp_{b_2}(gx, F(x_n,y_n,z_n)) + qp_{b_2}(gy, F(y_n,z_n,x_n)) + qp_{b_2}(gz, F(z_n,x_n,y_n))] \\
&\quad = k_1[qp_{b_2}(gx_n, gx) + qp_{b_2}(gy_n, gy) + qp_{b_2}(gz_n, gz)] \\
&\qquad + k_2[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1}) + qp_{b_2}(gz_n, gz_{n+1})] \\
&\qquad + k_3[qp_{b_2}(gx, F(x,y,z)) + qp_{b_2}(gy, F(y,z,x)) + qp_{b_2}(gz, F(z,x,y))] \\
&\qquad + k_4[qp_{b_2}(gx_n, F(x,y,z)) + qp_{b_2}(gy_n, F(y,z,x)] + qp_{b_2}(gz_n, F(z,x,y))] \\
&\qquad + k_5[qp_{b_2}(gx, gx_{n+1}) + qp_{b_2}(gy, gy_{n+1}) + qp_{b_2}(gz, gz_{n+1})] \\
&\quad \le k_1[qp_{b_1}(gx_n, gx) + qp_{b_1}(gy_n, gy) + qp_{b_1}(gz_n, gz)] \\
&\qquad + k_2[qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1})] \\
&\qquad + k_3[qp_{b_1}(gx, F(x,y,z)) + qp_{b_1}(gy, F(y,z,x)) + qp_{b_1}(gz, F(z,x,y))] \\
&\qquad + k_4[qp_{b_1}(gx_n, F(x,y,z)) + qp_{b_1}(gy_n, F(y,z,x)) + qp_{b_1}(gz_n, F(z,x,y))] \\
&\qquad + k_5[qp_{b_1}(gx, gx_{n+1}) + qp_{b_1}(gy, gy_{n+1}) + qp_{b_1}(gz, gz_{n+1})] \,.
\end{aligned}$$

Letting $n \to \infty$ in the above inequality, using (2.12)-(2.14) we obtain

$$\begin{aligned}
&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1}, F(x,y,z)) + qp_{b_1}(gy_{n+1}, F(y,z,x)) + qp_{b_1}(gz_{n+1}, F(z,x,y))] \\
&\quad \le \lim_{n\to\infty}\{k_1[qp_{b_1}(gx_n, gx) + qp_{b_1}(gy_n, gy) + qp_{b_1}(gz_n, gz)] \\
&\qquad + k_2[qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1}) + qp_{b_1}(gz_n, gz_{n+1})] \\
&\qquad + k_3[qp_{b_1}(gx_n, F(x,y,z)) + qp_{b_1}(gy, F(y,z,x)) + qp_{b_1}(gz, F(z_n,x,y))] \\
&\qquad + k_4[qp_{b_1}(gx_n, F(x,y,z)) + qp_{b_1}(gy_n, F(y,z,x)) + qp_{b_1}(gz_n, F(z,x,y))] \\
&\qquad + k_5[qp_{b_1}(gx, gx_{n+1}) + qp_{b_1}(gy, gy_{n+1}) + qp_{b_1}(gz, gz_{n+1})]\} \,.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1}, F(x,y,z)) + qp_{b_1}(gy_{n+1}, F(y,z,x)) + qp_{b_1}(gz_{n+1}, F(z,x,y))] \\
&\quad \le \{k_1[qp_{b_1}(gx, gx) + qp_{b_1}(gy, gy) + qp_{b_1}(gz, gz)] \\
&\qquad + k_2[qp_{b_1}(gx, gx)) + qp_{b_1}(gy, gy) + qp_{b_1}(gz, gz)] \\
&\qquad + k_3[qp_{b_1}(gx, F(x,y,z)) + qp_{b_1}(gy, F(y,z,x)) + qp_{b_1}(gz, F(z,x,y))] \\
&\qquad + \lim_{n\to\infty} k_4[qp_{b_1}(gx_n, F(x,y,z)) + qp_{b_1}(gy_n, F(y,z,x)) + qp_{b_1}(gz_n, F(z,x,y))] \\
&\qquad + k_5[qp_{b_1}(gx, gx) + qp_{b_1}(gy, gy) + qp_{b_1}(gz, gz)] \\
&\quad = k_3[qp_{b_1}(gx, F(x,y,z)) + qp_{b_1}(gy, F(y,z,x)) + qp_{b_1}(gz, F(z,x,y))] \\
&\qquad + \lim_{n\to\infty} k_4[qp_{b_1}(gx_n, F(x,y,z)) + qp_{b_1}(gy_n, F(y,z,x)) + qp_{b_1}(gz_n, F(z,x,y))];
\end{aligned}$$

By using (2.15)-(2.17), we get

$$\begin{aligned}&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1},F(x,y,z))+qp_{b_1}(gy_{n+1},F(y,z,x))+qp_{b_1}(gz_{n+1},F(z,x,y))]\\&\le k_3[qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x))+qp_{b_1}(gz,F(z,x,y))]\\&\quad+k_4\cdot s[qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x))+qp_{b_1}(gz,F(z,x,y))]\\&=(k_3+sk_4)[qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x))+qp_{b_1}(gz,F(z,x,y))];\end{aligned}$$

And also using (2.15)-(2.17) we get

$$\begin{aligned}&\frac{1}{s}[qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x)+qp_{b_1}(gz,F(z,x,y))]\\&\quad\le(k_3+sk_4)[qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x)+qp_{b_1}(gz,F(z,x,y))]\\\Rightarrow\quad&\left[\frac{1}{s}-k_3-sk_4\right][qp_{b_1}(gx,F(x,y,z))+qp_{b_1}(gy,F(y,z,x)+qp_{b_1}(gz,F(z,x,y))]\le 0\,.\end{aligned}\tag{2.18}$$

It follows from (2.1) that

$$k_3+sk_4<\frac{1}{s}.$$

Hence it follows from (2.18) that

$$qp_{b_1}(gx,F(x,y,z))=qp_{b_1}(gy,F(y,z,x))=qp_{b_1}(gz,F(z,x,y))=0\,.$$

By Lemma 1.3, we get $F(x,y,z)=gx$, $F(y,z,x)=gy$, $F(z,x,y)=gz$. Hence (gx,gy,gz) is a tripled point of coincidence of mappings F and g.

Next, we will show that the tripled point of coincidence is unique.

Suppose that $(x^*,y^*,z^*)\in X^3$ with $F(x^*,y^*,z^*)=gx^*$, $F(y^*,z^*,x^*)=gy^*$, and $F(z^*,x^*,y^*)=gz^*$. Using (2.2), (2.12), (2.13), (2.14) and ($\mathrm{QP_{b_3}}$), we obtain

$$\begin{aligned}&qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)+qp_{b_1}(gz,gz^*)\\&=qp_{b_1}(F(x,y,z),F(x^*,y^*,z^*))+qp_{b_1}(F(y,z,x),F(y^*,z^*,x^*))+qp_{b_1}(F(z,x,y),F(z^*,x^*,y^*))\\&\le k_1[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)+qp_{b_2}(gz,gz^*)]\\&\quad+k_2[qp_{b_2}(gx,F(x,y,z))+qp_{b_2}(gy,F(y,z,x))+qp_{b_2}(gz,F(z,x,y))]\\&\quad+k_3[qp_{b_2}(gx^*,F(x^*,y^*,z^*))+qp_{b_2}(gy^*,F(y^*,z^*,x^*))+qp_{b_2}(gz^*,F(z^*,x^*,y^*))]\\&\quad+k_4[qp_{b_2}(gx,F(x^*,y^*,z^*))+qp_{b_2}(gy,F(y^*,z^*,x^*))+qp_{b_2}(gz^*,F(z^*,x^*,y^*))]\\&\quad+k_5[qp_{b_2}(gx^*,F(x,y,z))+qp_{b_2}(gy^*,F(y,z,x))+qp_{b_2}(gz^*,F(z,x,y))]\\&=k_1[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)+qp_{b_2}(gz,gz^*)]\\&\quad+k_2[qp_{b_2}(gx,gx)+qp_{b_2}(gy,gy)+qp_{b_2}(gz,gz)]\\&\quad+k_3[qp_{b_2}(gx^*,gx^*)+qp_{b_2}(gy^*,gy^*)+qp_{b_2}(gz^*,gz^*)]\\&\quad+k_4[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)+qp_{b_2}(gz,gz^*)]\\&\quad+k_5[qp_{b_2}(gx^*,gx)+qp_{b_2}(gy^*,gy)+qp_{b_2}(gz^*,gz)]\\&\le(k_1+k_4)[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)+qp_{b_1}(gz,gz^*)]\\&\quad+k_2[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)+qp_{b_1}(gz,gz)]\\&\quad+k_3[qp_{b_1}(gx^*,gx^*)+qp_{b_1}(gy^*,gy^*)+qp_{b_1}(gz^*,gz^*)]\\&\quad+k_5[qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)+qp_{b_1}(gz^*,gz)]\\&\le(k_1+k_3+k_4)[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)+qp_{b_1}(gz,gz^*)]\\&\quad+k_5[qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)+qp_{b_1}(gz^*,gz)]\,.\end{aligned}$$

This implies that

$$qp_{b_1}(gx, gx^*) + qp_{b_1}(gy, gy^*) + qp_{b_1}(gz, gz^*)$$
$$\leq \frac{k_5}{1 - k_1 - k_3 - k_4}[qp_{b_1}(gx^*, gx) + qp_{b_1}(gy^*, gy) + qp_{b_1}(gz^*, gz)]\,. \tag{2.19}$$

Similarly, we have

$$qp_{b_1}(gx^*, gx) + qp_{b_1}(gy^*, gy) + qp_{b_1}(gz^*, gz)]$$
$$\leq \frac{k_5}{1 - k_1 - k_3 - k_4}[qp_{b_1}(gx, gx^*) + qp_{b_1}(gy, gy^*) + qp_{b_1}(gz, gz^*)]\,. \tag{2.20}$$

Substituting (2.20) into (2.19), we obtain

$$qp_{b_1}(gx, gx^*) + qp_{b_1}(gy, gy^*) + qp_{b_1}(gz, gz^*)$$
$$\leq \left(\frac{k_5}{1 - k_1 - k_3 - k_4}\right)^2 [qp_{b_1}(gx, gx^*) + qp_{b_1}(gy, gy^*) + qp_{b_1}(gz, gz^*)]\,. \tag{2.21}$$

Since $\frac{k_5}{1 - k_1 - k_2 - k_4} < 1$, from (2.21) we must have

$$qp_{b_1}(gx, gx^*) = qp_{b_1}(gy, gy^*) = qp_{b_1}(gz, gz^*) = 0\,.$$

By Lemma 1.3, we get $gx = gx^*$, $gy = gy^*$, and $gz = gz^*$ which implies that the uniqueness of the tripled point of coincidence of F and g, that is, (gx, gy, gz).

Next, we will show that $gx = gy = gz$. In fact, from (2.2), (2.12)-(2.14) we have

$$\begin{aligned}
&qp_{b_1}(gx, gy) + qp_{b_1}(gy, gz) + qp_{b_1}(gz, gx)\\
&= qp_{b_1}(F(x, y, z), F(y, z, x)) + qp_{b_1}(F(y, z, x), F(z, x, y) + qp_{b_1}(F(z, x, y), F(x, y, z))\\
&\leq k_1[qp_{b_2}(gx, gy) + qp_{b_2}(gy, gz) + qp_{b_2}(gz, gx)]\\
&\quad + k_2[qp_{b_2}(gx, F(x, y, z)) + qp_{b_2}(gy, F(y, z, x)) + qp_{b_2}(gz, F(z, x, y))]\\
&\quad + k_3[qp_{b_2}(gy, F(y, z, x)) + qp_{b_2}(gz, F(z, x, y)) + qp_{b_2}(gx, F(x, y, z))]\\
&\quad + k_4[qp_{b_2}(gx, F(y, z, x)) + qp_{b_2}(gy, F(z, x, y)) + qp_{b_2}(gz, F(x, y, z))]\\
&\quad + k_5[qp_{b_2}(gy, F(x, y, z)) + qp_{b_2}(gz, F(y, z, x)) + qp_{b_2}(gx, F(z, x, y))]\\
&= k_1[qp_{b_2}(gx, gy) + qp_{b_2}(gy, gz) + qp_{b_2}(gz, gx)]\\
&\quad + k_2[qp_{b_2}(gx, gx) + qp_{b_2}(gy, gy) + qp_{b_2}(gz, gz)]\\
&\quad + k_3[qp_{b_2}(gy, gy) + qp_{b_2}(gz, gz) + qp_{b_2}(gx, gx)]\\
&\quad + k_4[qp_{b_2}(gx, gy) + qp_{b_2}(gy, gz) + qp_{b_2}(gz, gx)]\\
&\quad + k_5[qp_{b_2}(gy, gx) + qp_{b_2}(gz, gy) + qp_{b_2}(gx, gz)]\\
&\leq k_1[qp_{b_1}(gx, gy) + qp_{b_1}(gy, gz) + qp_{b_1}(gz, gx)]\\
&\quad + k_2[qp_{b_1}(gx, gx) + qp_{b_1}(gy, gy) + qp_{b_1}(gz, gz)]\\
&\quad + k_3[qp_{b_1}(gy, gy) + qp_{b_1}(gz, gz) + qp_{b_1}(gx, gx)]\\
&\quad + k_4[qp_{b_1}(gx, gy) + qp_{b_1}(gy, gz) + qp_{b_1}(gz, gx)]\\
&\quad + k_5[qp_{b_1}(gy, gx) + qp_{b_1}(gz, gy) + qp_{b_1}(gx, gz)]\\
&= (k_1 + k_4)[qp_{b_1}(gx, gy) + qp_{b_1}(gy, gz) + qp_{b_1}(gz, gx)]\\
&\quad + k_5[qp_{b_1}(gy, gx) + qp_{b_1}(gz, gy) + qp_{b_1}(gx, gz)];
\end{aligned}$$

This implies that

$$qp_{b_1}(gx, gy) + qp_{b_1}(gy, gz) + qp_{b_1}(gz, gx)$$
$$\leq \frac{k_5}{1 - k_1 - k_4}[qp_{b_1}(gy, gx) + qp_{b_1}(gz, gy) + qp_{b_1}(gx, gz)]\,. \tag{2.22}$$

By similar arguments as above, we can show that

$$qp_{b_1}(gy,gx)+qp_{b_1}(gz,gy)+qp_{b_1}(gx,gz) \\ \leq \frac{k_5}{1-k_1-k_4}\left[qp_{b_1}(gx,gy)+qp_{b_1}(gy,gz)+qp_{b_1}(gz,gx)\right]. \tag{2.23}$$

Substituting (2.23) into (2.22), we have

$$qp_{b_1}(gx,gy)+qp_{b_1}(gy,gz)+qp_{b_1}(gz,gx) \\ \leq \left(\frac{k_5}{1-k_1-k_4}\right)^2\left[qp_{b_1}(gx,gy)+qp_{b_1}(gy,gz)+qp_{b_1}(gz,gx)\right]. \tag{2.24}$$

Since $\frac{k_5}{1-k_1-k_4}<1$, from (2.24), we must have

$$qp_{b_1}(gx,gy)=qp_{b_1}(gy,gz)=qp_{b_1}(gz,gx)=0\,.$$

By Lemma 1.3, we get $gx=gy=gz$.

Finally, assume that F and g are w-compatible. Let $u=gx$, then we have $u=gx=F(x,y,z)=gy=F(y,z,x)=gz=F(z,x,y)$, and so that

$$gu=ggx=g(F(x,y,z))=F(gx,gy,gz)=F(u,u,u)\,. \tag{2.25}$$

Consequently, (u,u,u) is a tripled coincidence point of F and g, and so (gu,gu,gu) is a tripled point of coincidence of F and g, and by its uniqueness, we get $gu=gx$. Thus we obtain $F(u,u,u)=gu=u$. Therefore, (u,u,u) is the unique common tripled fixed point of F and g. This complete the proof. □

Remark 2.2. *Theorem 2.1 improves and extends the main theorem of Gu [41] in the following aspects:*

(1) *The two quasi-partial metric extends to two quasi-partial b-metrics.*

(2) *The tripled fixed point in quasi-partial metric extends to a tripled fixed point in quasi-partial b-metric.*

In Theorem 2.1, if we take $qp_{b_1}(x,y)=qp_{b_2}(x,y)$ for all $x,y\in X$, then we get the following.

Corollary 2.3. *Let (X,qp_b) be a quasi-partial b-metric space, $F:X^3\to X$ and $g:X\to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4 and k_5 in $[0,1)$ with $k_1+k_2+k_3+2sk_4+k_5<\frac{1}{s}$ such that the condition*

$$\begin{aligned} &qp_b(F(x,y,z),F(u,v,w))+qp_b(F(gx,F(v,w,u))+qp_b(F(z,x,y),F(w,u,v)) \\ &\leq k_1[qp_b(gx,gu)+qp_b(gy,gv)+qp_b(gz,gw)] \\ &\quad+k_2[qp_b(gx,F(x,y,z))+qp_b(gy,F(y,z,x))+qp_b(gz,F(z,x,y))] \\ &\quad+k_3[qp_b(gu,F(u,v,w))+qp_b(gv,F(v,w,u))+qp_b(gw,F(w,u,v))] \\ &\quad+k_4[qp_b(gx,F(u,v,w))+qp_b(gy,F(v,w,u))+qp_b(gz,F(w,u,v))] \\ &\quad+k_5[qp_b(gu,F(x,y,z))+qp_b(gv,F(y,z,x))+qp_b(gw,F(z,x,y))] \end{aligned} \tag{2.26}$$

holds for all $x,y,z,u,v,w\in X$. Also, suppose we have the following hypotheses:

(i) *$F(X^3)\subset g(X)$;*

(ii) *$g(X)$ is a complete subspace of X.*

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = F(z, x, y) = gz\,.$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form $(u, u, u,)$.

The proof follows from Theorem 2.1. □

Corollary 2.4. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X such that $qp_{b_2}(x, y) \leq qp_{b_1}(x, y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exist $a_i \in [0, 1)$ $(i = 1, 2, 3, \ldots, 15)$ with*

$$\left(\sum_{i=1}^{9} a_i\right) + 2s\left(\sum_{i=10}^{12} a_i\right) + \left(\sum_{i=13}^{15} a_i\right) < \frac{1}{s} \tag{2.27}$$

such that the condition

$$\begin{aligned}
&qp_{b_1}(F(x, y, z), F(u, v, w))\\
&\quad \leq a_1 qp_{b_2}(gx, gu) + a_2 qp_{b_2}(gy, gu) + a_3 qp_{b_2}(gz, gw)\\
&\qquad + a_4 qp_{b_2}(gx, F(x, y, z)) + a_5 qp_{b_2}(gy, F(y, z, x)) + a_6 qp_{b_2}(gz, F(z, x, y))\\
&\qquad + a_7 qp_{b_2}(gu, F(u, v, w)) + a_8 qp_{b_2}(gv, F(v, w, u)) + a_9 qp_{b_2}(gw, F(w, u, v))\\
&\qquad + a_{10} qp_{b_2}(gx, F(u, v, w)) + a_{11} qp_{b_2}(gy, F(v, w, u)) + a_{12} qp_{b_2}(gz, F(w, u, v))\\
&\qquad + a_{13} qp_{b_2}(gu, F(x, y, z)) + a_{14} qp_{b_2}(gv, F(y, z, x)) + a_{15} qp_{b_2}(gw, F(z, x, y))
\end{aligned} \tag{2.28}$$

holds for all $x, y, z, u, v, w \in X$. Also suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to quasi-partial b-metric qp_{b_1}.*

(iii) *Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.*

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Proof. Given $x, y, z, u, v, w \in X$. It follows from (2.28) that

$$\begin{aligned}
&qp_{b_1}(F(x, y, z), F(u, v, w))\\
&\quad \leq a_1 qp_{b_2}(gx, gu) + a_2 qp_{b_2}(gy, gv) + a_3 qp_{b_2}(gz, gw)\\
&\qquad + a_4 qp_{b_2}(gx, F(x, y, z)) + a_5 qp_{b_2}(gy, F(y, z, x)) + a_6 qp_{b_2}(gz, F(z, x, y))\\
&\qquad + a_7 qp_{b_2}(gu, F(u, v, w)) + a_8 qp_{b_2}(gv, F(v, w, u)) + a_9 qp_{b_2}(gw, F(w, u, v))\\
&\qquad + a_{10} qp_{b_2}(gx, F(u, v, w)) + a_{11} qp_{b_2}(gy, F(v, w, u)) + a_{12} qp_{b_2}(gz, F(w, u, v))\\
&\qquad + a_{13} qp_{b_2}(gu, F(x, y, z)) + a_{14} qp_{b_2}(gv, F(y, z, x)) + a_{15} qp_{b_2}(gw, F(z, x, y)),
\end{aligned} \tag{2.29}$$

$$\begin{aligned}
&qp_{b_1}(F(y, z, x), F(v, w, u))\\
&\quad \leq a_1 qp_{b_2}(gy, gv) + a_2 qp_{b_2}(gz, gw) + a_3 qp_{b_2}(gx, gu)\\
&\qquad + a_2 qp_{b_2}(gy, F(y, z, x)) + a_5 qp_{b_2}(gz, F(z, x, y)) + a_6 qp_{b_2}(gx, F(x, y, z))\\
&\qquad + a_7 qp_{b_2}(gv, F(v, w, u)) + a_8 qp_{b_2}(gw, F(w, u, v)) + a_9 qp_{b_2}(gu, F(u, v, w))\\
&\qquad + a_{10} qp_{b_2}(gy, F(v, w, u)) + a_{11} qp_{b_2}(gz, F(w, u, v)) + a_{12} qp_{b_2}(gx, F(u, v, w))\\
&\qquad + a_{13} qp_{b_2}(gv, F(y, z, x)) + a_{14} qp_{b_2}(gw, F(z, x, y)) + a_{15} qp_{b_2}(gu, F(x, y, z))
\end{aligned} \tag{2.30}$$

and

$$\begin{aligned} &qp_{b_1}(F(z,x,y),F(w,u,v))\\ &\quad\le a_1qp_{b_2}(gz,gw)+a_2qp_{b_2}(gx,gu)+a_3qp_{b_2}(gy,gv)\\ &\quad+a_4qp_{b_2}(gz,F(z,x,y))+a_5qp_{b_2}(gx,F(x,y,z))+a_6qp_{b_2}(gy,F(y,z,x))\\ &\quad+a_7qp_{b_2}(gw,F(w,u,v))+a_8qp_{b_2}(gu,F(u,v,w))+a_9qp_{b_2}(gv,F(v,w,u))\\ &\quad+a_{10}qp_{b_2}(gz,F(w,u,v))+a_{11}qp_{b_2}(gx,F(u,v,w))+a_{12}qp_{b_2}(gy,F(v,w,u))\\ &\quad+a_{13}qp_{b_2}(gw,F(z,x,y))+a_{14}qp_{b_2}(gu,F(x,y,z))+a_{15}qp_{b_2}(gv,F(y,z,x)). \end{aligned} \tag{2.31}$$

Adding inequality (2.29) and (2.30) to inequality (2.31), we get

$$\begin{aligned} &qp_{b_1}(F(x,y,z),F(u,v,w))+qp_{b_1}(F(y,z,x),F(v,w,u))+qp_{b_1}(F(z,x,y),F(w,u,v))\\ &\quad\le(a_1+a_2+a_3)[qp_{b_2}(gx,gu)+qp_{b_2}(gy,gv)+qp_{b_2}(gz,gw)]\\ &\qquad+(a_4+a_5+a_6)[qp_{b_2}(gx,F(x,y,z))+qp_{b_2}(gy,F(y,z,x)+qp_{b_2}(gz,F(z,x,y))]\\ &\qquad+(a_7+a_8+a_9)[qp_{b_2}(gu,F(u,v,w))+qp_{b_2}(gv,F(v,w,u))+qp_{b_2}(gw,F(w,u,v))]\\ &\qquad+(a_{10}+a_{11}+a_{12})[qp_{b_2}(gx,F(u,v,w))+qp_{b_2}(gy,F(v,w,u))+qp_{b_2}(gz,F(w,u,v))]\\ &\qquad+(a_{13}+a_{14}+a_{15})[qp_{b_2}(gu,F(x,y,z))+qp_{b_2}(gv,F(y,z,x))+qp_{b_2}(gw,F(z,x,y))]. \end{aligned} \tag{2.32}$$

Proof. Put $(a_1+a_2+a_3)=k_1$, $(a_4+a_5+a_6)=k_2$, $(a_7+a_8+a_9)=k_3$; $(a_{10}+a_{11}+a_{12})=k_4$, $(a_{13}+a_{14}+a_{15})=k_5$ and then the result follows from Theorem 2.1.

Corollary 2.5. Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X such that $qp_{b_2}(x,y)\le qp_{b_1}(x,y)$, for all $x,y\in X$, and $F:X^3\to X$, $g:X\to X$ be two mappings. Suppose that there exists $k\in\left[0,\dfrac{1}{s}\right)$ such that the condition

$$\begin{aligned} &qp_{b_1}(F(x,y,z),F(u,v,w))+qp_{b_1}(F(y,z,x),F(v,w,u))+qp_{b_1}(F(z,x,y),F(w,u,v))\\ &\quad\le k[qp_{b_2}(gx,gu)+qp_{b_2}(gy,gv)+qp_{b_2}(gz,gw)] \end{aligned}$$

holds for all $x,y,z,u,v,w\in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3)\subset g(X)$.

(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.

Then the mappings F and g have a tripled coincidence point (x,y,z) satisfying $gx=F(x,y,z)=gy=F(y,z,x)=gz=F(z,x,y)$.

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u,u,u).

Proof. By putting $k_1=k$ and $k_2=k_3=k_4=k_5=0$ in Theorem 2.1 we get the desired result. □

Corollary 2.6. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X such that $qp_{b_2}(x,y)\le qp_{b_1}(x,y)$, for all $x,y\in X$, and $F:X^3\to X$, $g:X\to X$ be two mappings. Suppose that there exists $k\in\left[0,\dfrac{1}{s}\right)$ such the condition*

$$\begin{aligned} &qp_{b_1}(F(x,y,z),F(u,v,w))+qp_{b_1}(F(y,z,x),F(v,w,u))+qp_{b_1}(F(z,x,y),F(w,u,v))\\ &\quad\le k[qp_{b_2}(gx,F(x,y,z))+qp_{b_2}(gy,F(y,z,x))+qp_{b_2}(F(gz,F(z,x,y)))] \end{aligned}$$

holds for all $x,y,z,u,v,w\in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3)\subset g(X)$

(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial b-metric* qp_{b_1}.

Then the mappings F *and* g *have a tripled coincidence point* (x, y, z) *satisfying* $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Proof. By putting $k_2 = k$ and $k_1 = k_3 = k_4 = k_5 = 0$ in Theorem 2.1 we get the desired result. □

Corollary 2.7. *Let* qp_{b_1} *and* qp_{b_2} *be two quasi-partial b-metrics on* X *such that* $qp_{b_2}(x, y) \leq qp_{b_1}(x, y)$, *for all* $x, y \in X$, *and* $F : X^3 \to X$, $g : X \to X$ *be two mappings. Suppose that there exists* $k \in \left[0, \frac{1}{s}\right)$ *such the condition.*

$$\begin{aligned} &qp_{b_1}(F(x, y, z), F(u, v, w)) + qp_{b_1}(F(y, z, x), F(v, w, u)) + qp_{b_1}(F(z, x, y), F(w, u, v)) \\ &\quad \leq k[qp_{b_2}(gu, F(u, v, w)) + qp_{b_2}(gv, F(v, w, u)) + qp_{b_2}(gw, F(w, u, v))] \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. *Also, suppose we have the following hypotheses:*

(i) $F(X^3) \subset g(X)$.

(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial b-metric* qp_{b_1}.

Then the mappings F *and* g *have a tripled coincidence point* (x, y, z) *satisfying*

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F *and* g *are* w*-compatible, then* F *and* g *have a unique common tripled fixed point of the form* (u, u, u).

Proof. By putting $k_3 = k$ and $k_1 = k_2 = k_4 = k_5 = 0$ in Theorem 2.1 we get the desired result. □

Corollary 2.8. *Let* qp_{b_1} *and* qp_{b_2} *be two quasi-partial b-metric spaces on* X *such that* $qp_{b_2}(x, y) \leq qp_{b_1}(x, y)$, *for all* $x, y \in X$, *and* $F : X^3 \to X$, $g : X \to X$ *be two mappings. Suppose that there exists* $k \in \left[0, \frac{1}{2s^2}\right)$ *such that the condition*

$$\begin{aligned} &qp_{b_1}(F(x, y, z), F(u, v, w)) + qp_{b_1}(F(y, z, x), F(v, w, u)) + qp_{b_1}(F(z, x, y), F(w, u, v)) \\ &\quad \leq k[qp_{b_2}(gx, F(u, v, w)) + qp_{b_2}(gy, F(v, w, u)) + qp_{b_2}(gz, F(w, u, v))] \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. *Also, suppose we have the following hypotheses:*

(i) $F(X^3) \subset g(X)$

(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial b-metric* qp_{b_1}.

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Proof. By putting $k_4 = k$ and $k_1 = k_2 = k_3 = k_5 = 0$ in Theorem 2.1 we get the desired result. □

Corollary 2.9. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metric spaces on X such that $qp_{b_2}(x,y) \leq qp_{b_1}(x,y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings.*
Suppose that there exists $k \in \left[0, \frac{1}{s}\right)$ such that the condition

$$\begin{aligned} &qp_{b_1}(F(x,y,z), F(u,v,w)) + qp_{b_1}(F(y,z,x), F(v,w,u)) + qp_{b_1}(F(z,x,y), F(w,u,v)) \\ &\quad \leq k[qp_{b_2}(gu, F(x,y,z)) + qp_{b_2}(gv, F(y,z,x)) + qp_{b_2}(gw, F(z,x,y))] \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$

(ii) $g(X)$ is a complete subspace of X with respect to the quasi partial b-metric qp_{b_1}.
Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Proof. By putting $k_5 = k$ and $k_1 = k_2 = k_3 = k_4 = 0$ in Theorem 2.1 we get the desired result. ☐

Now, we introduce an example to support our results.

Example 2.10. *Let $X = [0, 1]$ and two quasi-partial b-metrics qp_{b_1} and qp_{b_2} on X be given as*

$$qp_{b_1}(x,y) = |x-y| + x \text{ and } qp_{b_2}(x,y) = \frac{1}{2}(|x-y| + x) \quad \text{for all } x, y \in X$$

with same coefficients $s = 1$. Also, define $F : X^3 \to X$ and $g : X \to X$ as

$$F(x,y,z) = \frac{x+y+z}{16} \quad \text{and} \quad g(x) = \frac{x}{2} \quad \text{for all } \ x, y, z \in X.$$

Then

(i) (X, qp_{b_1}) is a complete quasi-partial b-metric space.

(ii) $F(X^3) \subset g(X)$.

(iii) F and g are w-compatible.

(iv) For any $x, y, z, u, v, w \in X$, we have

$$\begin{aligned} &qp_{b_1}(F(x,y,z), F(u,v,w)) + qp_{b_1}(F(y,z,x), F(v,w,u)) + qp_{b_1}(F(z,x,y), F(w,u,v)) \\ &\quad \leq \frac{3}{8}(qp_{b_2}(gx, gu) + qp_{b_2}(gy, gv) + qp_{b_2}(gz, gw)). \end{aligned}$$

To verify (i) we proceed by observing that $qp_{b_1}(x,y) = |x-y| + x$ is a quasi-partial b-metric with $s = 1$. Hence a quasi-partial metric.

By Lemma 1.1, $(g(X), qp_{b_1})$ is complete if and only if $(g(X), p_{qp_{b_1}})$ is complete if and only if $(g(X), d_{p_{qp_{b_1}}})$ is complete.

Here,

$$\begin{aligned} p_{qp_{b_1}}(x,y) &= \frac{1}{2}[qp_{b_1}(x,y) + qp_{b_1}(y,x)] \quad \text{(Karapinar et al. [[10]])} \\ &= |x-y| + \frac{x+y}{2} \end{aligned}$$

and

$$\begin{aligned} d_{p_{qp_{b_1}}}(x,y) &= 2p_{qp_{b_1}}(x,y) - p_{qp_{b_1}}(x,x) - p_{qp_{b_1}}(y,y) \\ &= 2|x-y| + x + y - x - y \\ &= 2|x-y|. \end{aligned}$$

Clearly, $(g(X), d_{p_{qp_{b_1}}})$ is a complete metric space being a compact space.

To verify (ii).

Let $F(x,y,z)$ be an arbitrary element of $F(X^3)$.

We need to show that

$$F(x,y,z) = \frac{x+y+z}{16} \in g(X) = \left[0, \frac{1}{2}\right].$$

Since $x, y, z \in X = [0,1]$,

therefore $x+y+z \in [0,3]$ and hence $\dfrac{x+y+z}{16} \in \left[0, \dfrac{3}{16}\right] \subset \left[0, \dfrac{1}{2}\right]$.

Hence $F(X^3) \subset g(X)$.

The verification of (iii) is clear.

Now, we verify (iv)

For $x, y, z, u, v, w \in X$, we have

$$\begin{aligned} & qp_{b_1}(F(x,y,z)F(u,v,w)) + qp_{b_1}(F(y,z,x),F(v,w,u)) + qp_{b_1}(F(z,x,y),F(w,u,v)) \\ &= qp_{b_1}\left(\frac{x+y+z}{16}, \frac{u+v+w}{16}\right) + qp_{b_1}\left(\frac{y+z+x}{16}, \frac{v+w+u}{16}\right) \\ &\quad + qp_{b_1}\left(\frac{z+x+y}{16}, \frac{w+u+v}{16}\right) \\ &= \frac{1}{16}(|(x+y+z)-(u+v+w)| + (x+y+z) + |(y+z+x)-(v+w+u)| \\ &\quad + (y+z+x) + |(z+x+y)-(w+u+v)| + (z+x+y)) \\ &= \frac{3}{16}(|(x+y+z)-(u+v+w)| + (x+y+z)) \\ &\le \frac{3}{16}(|x-u| + |y-v| + |z-w| + x + y + z) \\ &= \frac{3}{8}\left(\left|\frac{x}{2}-\frac{u}{2}\right| + \left|\frac{y}{2}-\frac{v}{2}\right| + \left|\frac{z}{2}-\frac{w}{2}\right| + \frac{x}{2} + \frac{y}{2} + \frac{z}{2}\right) \\ &= \frac{3}{8}(qp_{b_2}(gx,gu) + qp_{b_2}(gy,gv) + qp_{b_2}(gz,gw)) \end{aligned}$$

Thus F and g satisfy all the hypotheses of Corollary 2.5. So, F and g have a unique common tripled fixed point. Here $(0,0,0)$ is the unique common tripled fixed point of F and g.

3 Conclusion

In this paper some new common tripled fixed point theorems for mappings defined on a set equipped with two quasi-partial b-metric spaces is proved with same coefficient s. The existing result in literature for coupled common fixed point on quasi-partial b-metric space is further generalized, improved and enriched in the present paper.

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Authors' contributions

Both authors contributed equally to this work. Both authors read and approved the final manuscript.

Competing Interest

The authors declare that they have no competing interests.

References

[1] Matthews SG. Partial metric topology. General Topology and its Applications, Ann. N.Y. Acad. Sci. 1994;728:183-197.

[2] Abdeljawad T, Karapinar E, Tas K. Existence and uniqueness of a common fixed point on partial metric spaces. Appl. Math. Lett. 2011;24(11):1900-1904.

[3] Abdeljawad T. Fixed points and generalized weakly contractive mappings in partial metric spaces. Math. Comput. Model . 2011;54(11-12):2923-2927.

[4] Altun I, Erduran A. Fixed point theorems for monotone mappings on partial metric spaces. Fixed Point Theory and Applications. 2011;508730.
DOI: 10.1155/ 2011/508730.

[5] Aydi H. Some fixed point results in ordered partial metric spaes. J. Nonlinear Sci. Appl. 2011;4(2):1-12

[6] Aydi H. Some coupled fixed point results on partial metric spaces. Int J. Math. Sci. 2011;647091.

[7] Chen C, Zhu C. Fixed point theorems for weakly C-contractive mappings in partial metric spaces. Fixed Point Theory and Applications. 2013;107.
DOI: 10.1186/1687-1812-2013-107.

[8] Karapinar E, Erhan I. Fixed point theorems for operators on partial metric spaces. Appl. Math. Lett. 2011;24:1894-1899.

[9] Haghi RH, Rezapour SH, Shahzad N. Be careful on partial metric fixed point results. Topol Appl. 2013;160:450-454.

[10] Karapinar E, Erhan I, Özürk A. Fixed point theorems on quasi-partial metric spaces. Math. Comput. Model. 2013;57:2442-2448.
DOI: 10.1016/j.mcm.2012.06.036.

[11] Bakhtin IA. The contraction principle in quasimetric spaces. It. Funct. Anal. 1989;30:26-37.

[12] Czerwik S. Contraction mappings in b-metric spaces. Acta Math. Inform. Univ. Ostrav. 1993;1:5-11.

[13] Shukla S. Partial b-metric spaces and fixed point theorems. Mediterranean Journal of Mathematics. 2013;11:703-711.
DOI: 10.1007/s00009-013-0327-4.

[14] Gupta A, Gautam P. Some coupled fixed point theorem on quasi-partial b-metric spaces. International Journal of Mathematical Analysis. 2015;9(6):293-306.
DOI:10.12988.ijma.

[15] Gupta A, Gautam P. Quasi-partial b-metric spaces and some related fixed point theorems. Fixed Point Theory and Applications. 2015;18.
DOI:10.1186/s13663-015-0260-2.

[16] Bhaskar TG, Lakshmikantham V. Fixed point theorems in partially ordered metric spaces and applications. Nonlinear Anal. 2006;65:1379-1393.

[17] Lakshmikantham V, Ćirić L. Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. Nonlinear Anal. 2009;70:4341-4349.

[18] Berinde V. Coupled coincidence point theorems for mixed monotone nonlinear operators. Comput. Math. Appl. 2012;64(6):1770-1777.

[19] Choudhary BS, Metiya N, Postolache M. A generalized weak contraction principle with applications to coupled coincidence point problems. Fixed Point Theory and Applications. 2013;152.
DOI: 10.1186/1687-1812-2013-152.

[20] Abbas M, Khan MA, Radenović S. Common coupled fixed point theorem in cone metric space for w-compatible mappings. Appl. Math. Comput. 2010;217:195-202.
DOI: 10.1016/j.amc.2010.05.042.

[21] Samet B, Vetro C. Coupled fixed point, f-invariant set and fixed point of N-order. Ann Funct. Anal. 2010;1(2):46-56.

[22] Aydi H, Abbas M. Tripled coincidence and fixed point results in partial metric spaces. Appl. Gen. Topol. 2012;13(2):193-206.

[23] Berinde V, Borcut M. Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces. Nonlinear Anal. 2011;74(15):4889-4897.

[24] Huang H. Radenović S, Vujaković J. On some recent coincidence and coupled coincidence point results in partially ordered b-metric spaces. Fixed point Theory and Applications. 2015;2015:63.

[25] Fadail ZM, Rad GS, Ozturk V, Radenović S. Some remarks on coupled, tripled and n-tupled fixed point theorems in ordered abstract metric spaces. Accepted in Far East Journal of Mathematical Sciences. 2015;97(7):809-839.

[26] Kadelburg Z, Kumam P, Radenović S, Sintunavarat W. Common coupled fixed point theorems for Geraghtyś type contraction mappings without mixed monotone property. Fixed Point Theory and Applications. 2015;2015:27.

[27] Rahimi H, Radenović S, Rad GS, Kumam P. Quadrupled fixed point results in abstract metric spaces, Comput. and Appl. Math.; 2013.
DOI.10.1007/S40314-013-088-5.

[28] Radenović S. A note on tripled coincidence and tripled common fixed point theorems in partially ordered metric spaces, Appl. Math. Comput. 2014;236:367-372.

[29] Alsulami HH, Roldan A, Karapinar E, Radenović S. Some inevitable remarks on 'Tripled fixed point theorems for mixed monotone kannan type contractive mappings'. Journal of Appl. Math; 2014. Article ID 392301, 7 pages.

[30] Agarwal RP, Kadelburg Z, Radenović S. On Coupled fixed point results in asymmetric G-metric spaces. Journal of Inequalities and Applications. 2013;2013:528.

[31] Long W, Shukla S, Radenović S, Radojević S. Some coupled coincidence and common fixed point results for hybrid pair of mappings in 0-complete partial metric spaces. Fixed Point Theory and Applications. 2013;2013:145.

[32] Parvaneh V, Roshan JR, Radenović S. Existence of tripled coincidence point in ordered b-metric spaces and applications to a system of integral equations. Fixed Point Theory and Applications. 2013;2013:130.

[33] Abbas M, Nazir T, Radenović S. Common coupled fixed point of generalized contractive mappings in partially ordered metric spaces. Positivity. 2013;17:1021-1041. DOI: 10.1007/S11117-012-0219-z.

[34] Sintunavarat W, Radenović S, Golubović Z, Kumam P. Coupled fixed point theorems for F-invariant set and applications. Appl. Math. Inf. Sci. 2013;7(1):247-255.

[35] Radenović S. Coupled fixed point theorems for monotone mappings in partially ordered metric spaces, Kragujevac Journal of Mathematics. 2014;38(2):249-257.

[36] Rajić VC, Radenović S. A note on tripled fixed point of w-compatible mappings in tvs-cone metric spaces. Thai Journal of Mathematics. 2014;12(3):717-728.

[37] Radenović S. Some coupled coincidence points results of monotone mappings in partially ordered metric spaces. Int. J. Anal. Appl. 2014;2(5):174-184.

[38] Radenović S. Bhaskar-Lakshmikantham type-results for monotone mappings in partially ordered metric spaces. Int. J. Nonlinear Anal. Appl. 2014;5(2):37-49.

[39] Aydi H. Abbas M, Sintunavarat W, Kumam P. Tripled fixed point of w-compatible mappings in abstract metric spaces. Fixed Point Theory and Applications; 2012. DOI: 10.1186/1687-1812-2012-134.

[40] Shatanawi W, Pitea A. Some coupled fixed point theorems in quasi-partial metric spaces. Fixed Point Theory and Applications; 2013. Article Id 153. DOI:10.1186/1687-1812-2013-153.

[41] Gu F. Some common tripled fixed point results in two quasi-partial metric spaces. Fixed Point Theory and Applications. 2014;71. DOI: 10.1186/1687-1812-2014-71.

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